SuccessMaker®

LEARNING LED BY YOU.

SuccessMaker Targeted Lessons

Multiplication and Division

Teacher's Resource Guide

SuccessMaker®

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Multiplication and Division

Teacher's Resource Guide

SAVVAS
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800-848-9500

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Introduction

Targeted Lessons Overview

These Targeted Lessons are designed to be used for small group or whole class intervention with students who are having difficulty with math topics in SuccessMaker, as shown on the Areas of Difficulty report. Lessons can also be adapted for use with individuals.

Targeted Lessons provide additional exposure to and focused instruction in specific math concepts and skills in order to deepen students' conceptual understanding and strengthen their performance. The lessons assume that students have had experience with the mathematics before; they are not intended as an initial introduction to a concept or skill. Some students may have learned a procedure without the necessary conceptual understanding that allows them to apply that procedure in new situations. Other students may have misconceptions that keep them from building on what they know or applying it accurately.

Targeted Lessons are unique in that they focus on finding and fixing student misconceptions. They use scaffolded, teacher-led instruction, carefully structured pair or small-group interaction, and guided mathematical discussion to build understanding.

Student Misconceptions

There is growing evidence that teaching is more effective when common mistakes and misconceptions are systematically exposed, challenged, and discussed. SuccessMaker Targeted Lessons are designed so that conflicts between mathematical beliefs or understandings and mathematical results are likely to occur. Such conflicts, when resolved through reflective discussion, lead to more permanent learning than conventional teaching methods that seek to avoid students making "mistakes."

Each lesson lists likely student misconceptions and common errors on the Overview page. The tasks and activities within a lesson that are likely to elicit a misconception are highlighted with this icon: ERROR

English Language Learners

SuccessMaker Targeted Lessons consider the academic, linguistic, and cultural challenges faced by students learning English as a second language and offer guidance to teachers. Focus on Language notes on the lesson Overview page list mathematical and academic language that teachers model during the lesson and all students are encouraged to use, including English language learners (ELLs). Spanish cognates are provided where appropriate. Throughout a lesson, ELLs are given a variety of scaffolded opportunities to talk and write about their mathematical thinking and learning. Look for specific English Language Learner notes to help you support your students during specific activities.

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Students with Special Needs

These lessons have been used successfully with students with special needs. By using the classroom routines, students are able to examine concepts in greater depth; they have more time to read, discuss, and explore strategies and answers. Within each lesson, you will find notes to help you support your students with special needs during specific activities.

Preparing to Use the Lessons

Identify Students Use the Areas of Difficulty report in SuccessMaker to identify students who are having difficulty with particular learning objects. Plan for a 30-minute time to meet with those students and teach the appropriate lesson.

Areas of Difficulty

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Math

Report Run: 07/20/17 - 12:43 PM

School: Independence Elementary School
Teacher: Erika Denoff
Grade: 2
Group: NA

Strand	Level	*Skill Description*		
		Student	Date at Risk	Targeted Lesson
Addition	1.10	*SMMA_LO_00015 - Add 1 to a number (sums 1 to 10).*		AS3. Adding Numbers from 1 to 20 Using the Commutative Property
		Travis Dantini	06/28/2017	
Addition	2.10	*SMMA_LO_00043 - Add three multiples of 10 (student choice, sums 30 to 90).*		AS9. Multiples of 10 That Add Up to 100
		Ladonna Kawliche	06/06/2017	
		Hua Vo	04/21/2017	
Addition	1.12	*SMMA_LO_00016 - Add two addends in words (one-digit addends, sums 6 to 10).*		AS3. Adding Numbers from 1 to 20 Using the Commutative Property
		Travis Dantini	04/05/2017	
Addition	2.05	*SMMA_LO_00044 - Add two multiples of 10 displayed horizontally (sums 20 to 90).*		
		Xavier Carruthers	04/04/2017	
		Ladonna Kawliche	07/11/2017	
Addition	1.05	*SMMA_LO_00014 - Add using basic math facts (addends 0 to 5, sums 1 to 5).*		AS1. Counting On and Back by 1s for Numbers 1 to 20
		Travis Dantini	07/12/2017	
		Nigel Rich	05/23/2017	

The Areas of Difficulty report shows which learning objects have appropriate Targeted Lessons for intervention.

Assign Partners or Small Groups In most lessons, students work in pairs or small groups, and it is often best if you form the groups or pairs rather than having students self-select. Pair students with similar math skill levels if possible. Pair English language learners with native English speakers or English language learners with similar levels of English language development.

Fold Show Me Cards Get the Show Me cards for the lesson and fold each so that you have teaching suggestions and answers on one side and the question for students on the other.

Get Materials for Students Be sure that each student has a "quick response board" (chalkboard, dry erase board, or several sheets of blank paper) and appropriate materials for writing large, dark letters and numbers. Students will hold up their answers to the Show Me questions so that you can quickly check answers. Be sure you have one set of the Student Practice pages for each student. Some lessons require manipulatives, scratch paper, scissors or other materials. Check the Materials Needed list on the lesson overview page.

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Learning Routines and Rituals

All Targeted Lessons follow a similar routine. A lesson includes time for using the Show Me Cards, a setting the direction activity, student work time, a teacher-facilitated discussion for probing for understanding, and a final student reflection writing prompt. Depending on the purpose of the activity, the Show Me part of the routine may come at the beginning or near the end of the lesson.

Activities within the lessons refer to rituals—the ways in which the work is accomplished. For example, in the Show Me ritual, you hold up the Show Me cards. Without discussion, students write their answers on a "quick response board," a dry erase board, chalkboard, or blank sheet of paper. They hold up their responses so that you can quickly check them visually.

While no student is likely to use all of the lessons, the consistent lesson routine and rituals help students focus on lesson content rather than on understanding what to do. This is particularly true for English language learners and students with special needs. You might decide to teach two or three of the lessons to the whole class initially, so that everyone becomes familiar with the routines and rituals. Then, when you use the lessons as intervention for students who are struggling with specific SuccessMaker learning objects, students will be able to work comfortably, with few questions about procedure.

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Ritual	Teacher and Student Responsibilities
Using the Show Me Cards	• Students answer Show Me Card problems without discussion • Students hold up responses when teacher says, "Show Me." • Show Me time is brief, 3–6 minutes. There may not be time to use all the Show Me Cards for a lesson.
Using the Student Practice Pages	• Students write with pen or marker. They cross out things they want to change, but do not erase. • Teachers review student pages and follow-up as needed.
Presenting the Task	• Teacher or student reads the task aloud. Two–four students explain the task in their own words. • Teachers focus on helping students understand the task, not solve the problem. • Students begin when teacher says, "Do it now."
Solo Work	• Students think and begin work quietly. They may not finish before the teacher asks them to work with a partner. • Students use drawings or manipulatives as needed. • If students get stuck, they write questions about things they do not understand.
Partner Work	• Each person shows and explains the work he or she began while working solo to the partner. • Partners compare how their work is the same and different. • Partners complete the tasks together, editing earlier work if needed.
Probing for Understanding	• Teacher selects students to explain solutions, including why students decided on their solution strategies. • Students ask questions of one another about the solutions. • Teacher uncovers misconceptions; clarifies thinking; investigates approaches that did and did not work; solidifies the concepts.

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Mathematical Content Overview

Lesson	Goals
1: Multiplication as Equal Groups	Make sets of equal groups, represent them by writing equations, and recognize that sets of equal groups are equivalent
2: Equal Groups in Context	Use math drawings and equations to find the total in multiplication situations
3: Multiplication as Arrays	Use rectangular arrays and equations to find the total in multiplication situations; represent an area situation with an equation
4: Multiplication Equations in Context	Develop a multiplication situation when given an equation and represent the situation in multiple ways
5: Comparison Situations	Identify the difference between additive comparison and multiplicative comparison situations
6: Writing Comparison Situations	Write word problems involving multiplicative comparison
7: Using Data from Graphs with Different Scales	Multiply to solve word problems involving data from pictographs and bar graphs with different scales
8: Representing Division	Recognize division situations in which the total amount and the number in each group are known
9: Another Kind of Division	Represent division situations in which the total amount and number of groups are known
10: Inverse Operations	Explore the inverse relationship between multiplication and division, using visual representations and equations
11: Using Multiplication to Solve Division Problems	Identify and correct errors in division problems, using estimation and multiplication
12: Connecting Representations in a Problem	Represent and solve word problems with multiplication and division situations
13: Multiplication and Division	Write and solve one-step multiplication and division word problems (equal groups) using equations with a letter standing for the unknown quantity
14: More Multiplication and Division Word Problems	Correct errors in one-step word problems represented in models and equations with a letter standing for the unknown quantity
15: One-Step Word Problems	Match one-step word problems using all four operations with diagrams, equations, and answers
16: Hidden Questions and First-Step Equations	Identify hidden questions and first-step equations for two-step word problems
17: Multiplying by Multiples of Ten	Multiply one-digit whole numbers by multiples of 10; identify and explains arithmetic patterns
18: How Much Is 15 Tens?	Solve problems with multiples of 10 by modeling with base-10 block and using number sense

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Lesson	Goals
19: How Many Tens?	Identify numerical expressions that represent sets of objects grouped in multiples of 10, using number sense and properties of operations
20: Using Equations to Multiply Tens	Multiply a multiple of 10, decomposing the multiplier and applying properties of operations and number sense
21: Times Tens Concentration	Identify representations related to multiplying one-digit whole numbers by multiples of 10 and multiples of 100
22: Solving Problems With Multiples of 10 and 100	Solve multiplication problems with multiples of 10 and 100
23: Counting and Calculating the Area of Rectangles	Use counting, repeated addition, and multiplication to find the area of rectangles
24: Matching Areas of Rectangles	Match the area of rectangles with diagrams and dimensions
25: Using Addition and Multiplication to Find the Area of Rectangles	Use addition and multiplication to find the area of rectangles
26: Solving Area Word Problems	Use multiplication to solve area word problems
27: Relating Area and Perimeter	Relate area and perimeter
28: Area and Perimeter Word Problems	Use addition and multiplication to solve area and perimeter word problems
29: The Distributive Property and Area	Connect the distributive property and area
30: Using the Distributive Property	Represent and use the distributive property with arrays and equations
31: Multiplying Multi-digit Numbers	Multiply multi-digit numbers
32: Multiplying by Two-Digit Numbers	Multiply two two-digit numbers
33: Relating Division to Multiplication	Relate multiplication to division to find quotients involving multi-digit dividends
34: Estimating When Dividing	Estimate the quotient in division problems
35: Interpreting Remainders	Correct student work involving the interpretation of remainders
36: Repeating Shape Patterns	Generate a repeating shape pattern that follows a given rule

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Multiplication as Equal Groups

1

Mathematical Goals

- Understand that multiplication is the operation that is used to join a number of sets (consisting of one thing or collections of things) of identical size
- Recognize that an equation consists of two expressions linked by an equals sign and that an equals sign means the amounts on the two sides of an sign are the same

Misconceptions Addressed

ERROR

- Does not understand the concept of equivalence
- Does not understand that all groups must be the same size when multiplying and dividing
- Is unable to transfer between different representations of the same operation

Materials Needed

- ✓ Student pages and markers
- ✓ Response boards
- ✓ Show Me Cards MD1-1 through MD1-6
- ✓ Counters (approximately 30 per student)
- ✓ Large sheet of paper and markers to record equations

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- diagram *(diagrama)*
- equation *(ecuación)*
- equal groups
- equivalent *(equivalente)*
- math drawing

english language learners

Developing and deepening a student's understanding of new mathematical words is essential for English language learners. Make a point of modeling and/or interacting with new vocabulary throughout each lesson. If appropriate, use graphic organizers and manipulatives to review mathematical concepts and vocabulary.

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show me

Use Show Me Cards **MD1-1** through **MD1-6**.

During today's show me, students will complete equations involving multiplication facts with 6 as a factor. Each card prompts students to write the answer as an equation. Make sure students know that an equation is a mathematical statement using symbols to show that two amounts are equal.

RESPONSE BOARDS

Ask students to take out their response boards. Have students answer the following questions on their response boards. (Note that the answers are on the backs of the cards.)

- **MD1-1** How much is 6 groups of 6? Show me your answer as an equation.
- **MD1-2** 42 is equal to 6 times what number?
- **MD1-3** 6 times what number is 54?
- **MD1-4** 6 times what number is 18?
- **MD1-5** How much is 6 times 4?
- **MD1-6** 6 times what number equals 0?

To finish, have students complete the show me problem on the student pages. Be sure to allow time for this final show me problem. If necessary, omit some of the designated Show Me Cards. Read the problem to students and make sure that everyone knows what to do. Then ask students to write their responses to the show me problem.

Multiplication as Equal Groups 1

show me

Solve.

6 × 8 = 48

Student page 1

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setting the direction

Introducing the Lesson

Tell students that the purpose of today's lesson is to make and describe equal groups, rearrange materials to make an equivalent set of equal groups, and record equations that use multiplication.

Explain that students are required to listen to all directions first. Tell them that you will use the phrase "Do it now," or "You may begin," to let them know when they can start working with manipulatives.

Use the examples in setting the direction to illustrate these procedures.

Ask questions such as the following:

- Let's look at <Name 1>'s grouping and <Name 2>'s grouping. How are they the same? How are they different?
- Where do we see the 12 in this grouping?
- How can we use equations to record the fact that any two of these groupings show the same number of counters?

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scaffolding for success

The equal groups are shown here in linear arrangements to save space. Student arrangements may look very different. As long as you can distinguish the correct number of equal groups, the physical arrangement is not important.

Record equations on a group chart like this:

12	
6 • 2 = 3 • 4	6 • 2 = 4 • 3
6 • 2 = 2 • 6	2 • 6 = 3 • 4
3 • 4 = 6 • 2	3 • 4 = 4 • 3

setting the direction

example

How many different ways can you use 12 counters to make equal groups? Sketch all the ways you can think of to make equal groups.

4 × 3 xxx xxx xxx xxx

3 × 4 xxxx xxxx xxxx

6 × 2 xx xx xx xx xx xx

2 × 6 xxxxxx xxxxxx

12 × 1 x x x x x x x x x x x x

1 × 12 xxxxxxxxxxxx

Write at least two equations that show that one set of equal groups is equivalent to a different set of equal groups.

4 × 3 = 6 × 2

4 × 3 = 3 × 4

Student page 1

work time

Presenting the Task

Read the directions for problem 1 aloud to students.

- I want you to do problems 1a and 1b working solo—that means working by yourself. If you finish before I ask you to stop, you can turn to a partner and compare your answers.
- You may begin.

work time

1. Use 18 counters to make equal groups.
   a. Sketch all the ways you can think of to make equal groups.

18 × 1 x x x x x x x x x x x x x x x x x x

9 × 2 xx xx xx xx xx xx xx xx xx

6 × 3 xxx xxx xxx xxx xxx xxx

3 × 6 xxxxxx xxxxxx xxxxxx

2 × 9 xxxxxxxxx xxxxxxxxx

1 × 18 xxxxxxxxxxxxxxxxxx

Student page 2

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Have students work individually on problems 1a and 1b. If time permits, you can allow students to compare their answers as informal partner work.

Remind students to use counting on and counting back to complete these exercises. Refer them to the example problems if they get stuck.

b. Write at least two equations that show that one set of equal groups is equivalent to a different set of equal groups.

Possible answers:

$9 \times 2 = 18 \times 1$ $\quad$ $3 \times 6 = 6 \times 3$

$2 \times 9 = 3 \times 6$ $\quad$ $1 \times 18 = 9 \times 2$

Student page 2

english language learners

Encourage ELL students to use their English speaking skills in class. At the same time allow for use of their native languages to clarify concepts during partner work. New ELLs may benefit from partners who speak the same native language.

probing for understanding

Ask questions similar to these:

- How are the groupings the same?
- How are they different?
- Where do we see the 18 in this grouping?
- How can we record this?
- Are the groupings equivalent?
- How would you describe the equal groups?

Allow students time to edit their answers on their student pages if they need to add details or want to add another sentence after hearing your questions and other students' explanations.

scaffolding for success

Equations are mathematical statements of equality that are written horizontally. Be sure that students understand what is meant by the term "equation," and check to see that they have written appropriate equations. If necessary, allow time for students to add equations to their work.

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work time

Presenting the Task

Tell students that sometimes you will read the problem aloud and sometimes the group will read the problem together. This problem you will read together.

Have students work individually on problems 2a and 2b.

2. a. Use 24 counters to make equal groups and sketch your groupings.

24 × 1	x x x x xxxxxxxxxxxxxxxxxxxx
12 × 2	xx xx xx xx xx xx xx xx xx xx xx xx
8 × 3	xxx xxx xxx xxx xxx xxx xxx xxx
6 × 4	xxxx xxxx xxxx xxxx xxxx xxxx
4 × 6	xxxxxx xxxxxx xxxxxx xxxxxx
3 × 8	xxxxxxxx xxxxxxxx xxxxxxxx
2 × 12	xxxxxxxxxxxx xxxxxxxxxxxx
1 × 24	xxxxxxxxxxxxxxxxxxxxxxxx

b. Write six equations that show that one of your sets of equal groups is equivalent to another set of equal groups.

Possible answers:

6 × 4 = 24 × 1	2 × 12 = 3 × 8
8 × 3 = 3 × 8	12 × 2 = 4 × 6
4 × 6 = 6 × 4	1 × 24 = 8 × 3

Student page 3

probing for understanding

Ask questions such as the following:

- Where do you see 24 in your counters?
  Answer: I see three groups of 8; I see four groups of 6.
- How can we see multiplication in your groupings?
  Answer: We can see the number of groups and we can see the number of counters in each group.
- How can we see equivalence in your groupings?
  Answer: Both groupings contain the same number of 24 counters.

Ask students to share their equations using multiplication about 24, and record them on a group chart.

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reflection

Modeling the Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished. Tell students how pleased you are with their work today.

Read the reflection prompt to students.

Have students respond to the reflection prompt.

reflection

Something new I learned today is …

Fill in the shaded squares in the multiplication table below.

×	0	1	2	3	4	5	6	7	8	9	10
0							0 × 6 0				
1							1 × 6 6				
2							2 × 6 12				
3							3 × 6 18				
4							4 × 6 24				
5							5 × 6 30				
6	6 × 0 0	6 × 1 6	6 × 2 12	6 × 3 18	6 × 4 24	6 × 5 30	6 × 6 36	6 × 7 42	6 × 8 48	6 × 9 54	6 × 10 60
7							7 × 6 42				
8							8 × 6 48				
9							9 × 6 54				
10							10 × 6 60				

Student pages 4–5

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Equal Groups in Context 2

Mathematical Goals

- Understand that multiplication situations can be represented in many ways including the use of concrete materials, math drawings, diagrams, and equations

Misconceptions Addressed

- Does not recognize a multiplication situation
- Does not deal with remainders in problem situations appropriately, including thinking remainders can be greater than divisors
- Does not understand the concept of equivalence

Materials Needed

- ✓ Student pages and markers
- ✓ Response boards
- ✓ Show Me Cards MD2-1 through MD2-6
- ✓ Counters (approximately 30 per student)
- ✓ Large sheet of paper and markers to record equations

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- multiply *(multiplicar)*
- multiplication *(multiplicación)*
- model (a multiplication situation)
- product
- repeated addition
- represent (a multiplication situation) *(representar)*
- tape diagram

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show me

RESPONSE BOARDS

Use Show Me Cards **MD2-1** through **MD2-6**. During today's show me, students will answer questions involving division by 6. Have students answer the following questions on their response boards.

- **MD2-1** Twelve divided by 6 equals what number? Show me your answer as an equation.
- **MD2-2** Six times what number is 30?
- **MD2-3** Six times what number equals 36?
- **MD2-4** Six divided by 6 equals what number?
- **MD2-5** What number divided by 6 equals 10?
- **MD2-6** What number divided by 6 equals 9?

To finish, have students complete the show me problem.

Equal Groups in Context 2

show me

What number makes this equation true?

42 ÷ 6 = 7

Student page 7

setting the direction

Introducing the Lesson

Tell students they are going to look at situations in which there are a number of groups and each group has an equal number of items. They will be using multiplication to find the total number of items.

Point out that in real-life situations, many different words or phrases can be used to describe these equal groups, such as boxes, bags, plates, shelves, purses, and so on.

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work time

Presenting the Task

Read the problem together. Tell students to think about what the problem is asking. They can also use any of the materials provided to help them think about the problem.

work time

1. Mrs. Chi is buying fruit for a class field trip. She wants to buy enough oranges to give 1 to each of her 29 students. Mrs. Chi is going to buy 5 bags of oranges. In each bag there are 6 oranges.

   Will Mrs. Chi have enough oranges for her students? Explain your thinking.

   5 × 6 = 30

   Yes, Mrs. Chi will have enough oranges for her students. She will have 30 oranges. 30 oranges is 1 more than 29.

Write an equation that you could use to find the answer.

Remember to write your answer as a complete sentence.

Student page 7

Modeling "Ask Myself" Questions

Model how to use "Ask Myself" questions using problem 1.

- If I looked at this problem, I would start by asking myself questions to help me understand the problem. Some questions I would ask are:
  - What is the problem talking about? Maybe I can restate the problem in my own words or I can ask you to help me do that.
  - Math problems usually have things we can count or measure, such as apples or money or lengths. What are the things I could count in this problem? It looks like those are students and bags of oranges and the number of oranges in a bag.
  - How many students are there in all? Where do I see that number in the problem? I see in the second sentence there are 29 students.
  - How many bags of oranges are there? Where do I see that number in the problem? I see in the third sentence there are 5 bags of oranges.
  - How many individual oranges are there in the problem? Where do I see that number in the problem? I don't know; that is what I am trying to find out.

Asking yourself these kinds of questions will help you solve the problem.

english language learners

"Ask Myself" questions are good meaning-making strategies that students can use as tools for approaching future learning challenges. These types of questions help English language learners help themselves during difficult language situations.

It would be helpful to students if you placed these "Ask Myself" questions on chart paper so that students can refer to them as needed.

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teaching strategies

Students might think the answer to the final question is the number 30 because this problem involves a fact that they already know. When you ask, "Where do you see that number in the problem?" they might not be able to find it. The answer may seem so obvious to students that they do not even realize that they "worked a problem" to get it. If this type of thinking occurs, ask them to consider the same problem with larger numbers, like 68 bags with 6 oranges in each bag for all 437 students in the school. If that were the problem, how would they answer the question? The goal is to understand problem structure, even for easy problems.

Modeling the Situation with Counters

- I will model this situation using counters. If I had oranges, I might try putting them in bags to act out the problem. Instead, I am going to use these counters to represent the oranges. How could I use them to model the situation?
  - How many counters would I need to represent the whole class? When I asked myself questions, I said there were 29 students in the class.
  - What does each of these counters represent? Can you help me answer that question?

  Answer: The counters could represent oranges or bags of oranges. It's probably easier to solve the problem if each counter represents 1 orange.

Before instructing students to take counters, remind them that they should always listen to all of the instructions before they begin working with manipulatives. They should wait for you to say, "Do it now," or "You may begin," which will be the signal that you are finished with instructions.

Point to the hints on page 7 of the student pages. Students should notice that it has reminders to write the answer as a complete sentence and to write an equation. Whenever students solve a math story, they should always write the answer as a complete sentence and write an equation because these are two elements of a complete solution.

scaffolding for success

Depending on the grade level of students, they may not understand what is meant by a complete sentence. You may need to model a complete sentence for them either now or during probing for understanding.

Have students solve the problem with the counters. Tell them to pay attention to the steps they take because you will talk about what they did later.

Emphasize that solo work time is quiet work time. Indicate that it is time to begin working by saying, "Do it now" or "You may begin."

Observe students as they work on the problem.

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probing for understanding

Ask questions similar to:

- Why did you make 5 groups with your counters?

  Answer: Mrs. Chi is buying 5 bags of oranges; that is 5 groups.

- How many counters were in each group? Why?

  Answer: There were 6 counters, because there were 6 oranges in each bag.

Introducing New Ideas: Equations

Look to see if anyone included an equation as part of their solution.

- I see that <Name> included an equation. An equation is a mathematical statement using symbols to show that two amounts are equal. The equals sign tells us that the amounts on each side of the equation are the same, even if they look different. What are the two amounts in your equation?
  Or
  I see lots of good math drawings that match this problem, but no one included an equation. An equation is a mathematical statement using symbols to show that two amounts are equal. The equals sign tells us that the amounts on each side of the equation are the same, even if they look different. Can anyone think of an equation that matches this problem? (The most likely equation will be $5 \times 6 = 30$.) Just like we try to include a math drawing with our solutions to math stories, we should also write an equation that matches the story. Add one to your solution now.

- Where do you see in your equation that Mrs. Chi will have enough oranges?

  Answer: In the product 30

work time

Presenting the Task

Have students read problem 2 quietly to themselves, then ask a student to restate the problem in their own words.

Indicate the different materials that can be used to help solve the problem. Ask students to think about which materials might be easiest to use to represent the situation.

2. Tran has a stamp album with 4 pages. There are 10 stamps on each page.

How many stamps does Tran have altogether?

4 × 10 = 40

Tran has 40 stamps altogether.

Write an equation that you could use to find the answer.

Remember to write your answer as a complete sentence.

Student page 8

Observe students as they work. Do not be too quick to offer additional help. Allow individual "struggle time."

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probing for understanding

Ask questions such as:

- <Name>, I noticed that you used base-10 blocks to represent the problem. Can you tell us what you did and how it helped you?
- <Name>, why did you put out four tens?

  Answer: There are 4 pages of 10 stamps.
- How were you able to use the base-10 blocks to find a solution?

  Answer: I counted by tens.

Introducing New Ideas: Column Form

Tell students that mathematicians use the term "column form" when they are talking about one method used to find the answer to arithmetic problems. They would call each of these an example of column form:

$$\begin{array}{r} 23 \\ 56 \\ +\ 17 \\ \hline 96 \end{array} \qquad \begin{array}{r} 56 \\ -\ 17 \\ \hline 39 \end{array} \qquad \begin{array}{r} 51 \\ \times\ 7 \\ \hline 357 \end{array} \qquad \begin{array}{r} 6 \\ 4\overline{)24} \\ -\ 24 \\ \hline 0 \end{array}$$

Write a multiplication problem in column form that can be used to solve problem 2 on the board and tell students to write a similar problem in their books at the same time.

work time

Presenting the Task

Have students listen while you read the problem aloud.

Introducing New Ideas: Math Drawings

Tell students a math drawing is like a picture that shows what you did but does not show details such as what each student looked like. Model sketching a math drawing like the one in the answer key.

3. There are 9 boats on the lake.
Each boat has 4 people in it.

How many people are in the boats on the lake?

Make a math drawing to show the people and boats.

Write an equation that you could use to find the answer.

Remember to write your answer as a complete sentence.

9 × 4 = 36

There are 36 people in the boats on the lake.

Student page 8

Observe students as they work to see whether they are able to represent the situation with a math drawing. Note those students who add too much detail. Math drawings are quick and easy. They show the relationships among the quantities and are labeled, but they are not very realistic looking. Note: Do not expect students to label this math drawing.

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Ask students to stop after 1–3 minutes of solo work and have them join with partners. Pair students with others who are at a similar stage of working on the task, such as students who have made math drawings but not found a solution. Ask the partners to compare their math drawings and work together to find a solution. If a pair of students has already made math drawings and found solutions, ask them to share the ways they used their math drawings to find a solution.

If students seem to be having difficulty showing the situation with a math drawing, help them by giving a new situation—changing the context but using the same numbers. For example: "Miguel baked some cookies. He put out 9 plates and on each plate placed 4 cookies." Then ask the students to make a math drawing showing the new context.

probing for understanding

Ask questions such as:

- Where do you see the number of boats on the lake in your math drawing?
- Where do you see the number of people in each boat in your math drawing?
- Look at <name>'s drawing and <name>'s drawing. How are they the same and how are they different?
- Does it matter that <name> uses dots to show the number of people in each boat, and <name> uses circles? Why not?
- How were you able to use the math drawing to find a solution?
- Did anyone write a problem in column form?

If no one used column form, ask students:

- What might the column form look like?
- Where do you see the number of boats in your column form problem?
  Answer: The number 9
- What does the 4 in your column form problem represent?
  Answer: The number of people in each boat
- How does your answer in the column form problem compare to the math drawing?
  Answer: The answer (36) matches the total number of people in the math drawing.

Allow students time to share their math drawings and explain their solutions.

Introduce the use of a tape diagram to represent a multiplication situation.

- Most of your diagrams use sketches that represent counters or other materials. Some diagrams use numbers and words to represent the problem. Tape diagrams fall into this category. Has anyone used a tape diagram before?

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- A tape diagram for multiplication has more sections than the tape diagrams for addition and subtraction typically have. If we were making a tape diagram for this problem, we would probably use the sections in the tape to represent the boats. How many sections (or rectangles) would we need?

  Answer: 9

Begin creating a tape diagram on a chart that will remain posted.

- Inside each rectangle, we would write the number of people in each boat. What number would we put into each section?

  Answer: 4

Add this to your tape diagram.

- To find the total number of people in the boats, we would add all the values in the sections of the tape, just like we do for an addition diagram: 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 = 36. We call this type of addition repeated addition because we add the same number again and again. Multiplication is a fast way to do repeated addition.

reflection

Writing a Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

reflection

Multiplying is ☺ easy for me ☹ hard for me (circle one)

because …

Student page 9

Today students are asked to respond to a sentence starter. Be sure that they understand that they do not have to respond only in words. Students can give an example of something they learned. For example, they can write a math fact that they learned, they can make a math drawing, or they can use a diagram. If students do decide to use words in their explanations, they can use invented spelling, just like they do when they write stories.

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Multiplication as Arrays 3

Mathematical Goals

- Use arrays or grids to represent multiplication situations
- Find the total number of items in an array by multiplying the number of rows by the number of items in each row
- Use equations to represent area situations

Misconceptions Addressed

- Is unable to transfer between different representations of the same operation
- Does not understand how repeated addition relates to multiplication or how repeated subtraction relates to division

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD3-1 through MD3-6
✓ Square tiles (at least 48 per student)
✓ Scissors and glue (one per student)

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- area *(área)*
- match
- rectangular array
- row

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards **MD3-1** through **MD3-6**. During today's show me, students will answer questions about multiplication facts involving the number 7. Have students answer the following questions on their response boards.

- **MD3-1** How much is 7 times 2? Show me your answer as an equation.
- **MD3-2** Seven times what number equals 35?
- **MD3-3** What number equals 7 times 7?
- **MD3-4** How much is 7 times 100?
- **MD3-5** How much is 7 times 8?
- **MD3-6** Twenty-eight equals what number times 4?

To finish, have students complete the show me problem.

Multiplication as Arrays 3

show me

How many ◯s? Write an equation that matches this drawing.

9 × 7 = 63

Student page 11

setting the direction

Introducing the Lesson

Tell students that today they are going to make rectangular shapes with square tiles and find the total number of tiles in each shape using multiplication.

Introducing Rectangular Arrays

Use square tiles to model the following problem.

- How many tiles did Amir use to make the rectangular shape?

  Answer: 20 square tiles

setting the direction

example

Amir made a rectangular shape using square tiles.
This array had 4 rows with 5 tiles in each row.

How many tiles did Amir use to make his rectangular array?

Amir used 20 tiles to make his rectangular array.

Student page 11

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Lay out a row of 5 tiles.

- How many tiles in this row?

  Answer: 5

Add the second row of 5 tiles.

- How many tiles do we have now?

  Answer: 10

Continue by placing the third and fourth rows of 5 tiles and repeating the questions each time.

- How many 5s did we add?

  Answer: 4

State that we can use a repeated addition equation to show how we found the number of tiles in the rectangular shape.

Write "5 + 5 + 5 + 5 = 20" on the board or paper.

- What multiplication equation can we use to show what we just did?

  Answer: $4 \times 5 = 20$

Add "$4 \times 5 = 20$" to your group chart of equations. Leave the chart posted as students work on problems 1 and 2.

Introducing New ideas: Answer as Part of a Complete Sentence.

- Since math story problems end with a question, we should answer that question using a complete sentence.

You may have to spend a few moments talking about what it means to answer the question using a complete sentence.

- What sentence could we use to answer the question in this math story?

  Answer: Amir used 20 tiles to make his rectangular array.

Add that sentence, or one that says the same thing, to your solution on the group chart.

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work time

Presenting the Task

Have students read problem 1 quietly to themselves and then restate the problem in their own words.

Have the container of tiles ready for students to begin work.

Ask students to begin work on problem 1.

As students finish, have them compare their responses with partners to make sure they have included all necessary representations and that their work makes sense.

1. Josh used some equal-sized squares to make a rectangular quilt. The quilt had 5 rows of squares, with 9 squares in each row.

   How many squares did Josh use to make his quilt? Use a rectangular array to solve this problem.

- Sketch your array.
- Write an equation that you could use to find the answer.
- Remember to write your answer as a complete sentence.

5 × 9 = 45

Josh used 45 squares to make his quilt.

Student page 12

Work with individuals or partners who need additional help.

Most students will not be used to including so much information in the answer to one problem. Remind them if they forget to include something. Be consistent in requiring that students include everything that the problem asks for.

probing for understanding

Allow time for students to share their arrays. Ask questions such as:

- Why were you able to solve this task using a multiplication equation?

  Answer: We knew the number of rows and how many squares were in each row.

- What other kind of equation could you write to match your array?

  Answer: Repeated addition

- Will both equations give the same answer?

  Answer: Yes

- Why?

  Answer: Because 9 added 5 times is the same as 5 groups of 9

english language learners

This type of partner work helps ELLs interact with other students and build their English speaking skills.

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work time

Presenting the Task

Tell students you will read the problem aloud and they should underline any words that they do not understand.

Students work individually to solve the problem.

Students work with partners to discuss and refine answers.

2. Tamika covered the front of her book with square tiles. She found that she made 6 rows with 8 tiles in each row. How many tiles did Tamika use altogether to cover her book? Use a rectangular array to solve this problem.

Suggestion: If you do not want to make a math drawing, use the grid paper on page 15 and cut out a model that matches your array. Glue it onto this page.

Remember to write your answer as a complete sentence.

6 × 8 = 48

Tamika used 48 tiles altogether to cover her book.

Student page 13

scaffolding for success

Encourage students to ask their partners to help clarify any concepts or terminology they still find confusing.

CENTIMETER GRID

Student page 15

probing for understanding

Ask questions such as:

- Where do you "see" the number of rows in your equation?

  Answer: The 6 is the number of rows.

- Where do you "see" the number of tiles in each row in your equation?

  Answer: The 8 is the number of tiles in each row.

- Can anyone tell us the shortcut for finding the number of objects in a rectangular array?

  Answer: Multiply the number of rows by the number of columns.

scaffolding for success

Be sure to point to the parts of the diagram as you discuss them.

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- The rows and columns in an array are a lot like the width and length of a rectangle. What do we get when we multiply the length times the width of a rectangle?

  Answer: The area

- How are those two ideas alike?

  Answer: The number of objects in a rectangular array is the same as the area of a rectangle with the same dimensions.

- How are they different?

  Answer: The differences are that:

  - Arrays are not always made up of square units so sometimes the objects in the array do not really cover the whole area.
  - Arrays can only be made using whole numbers of objects. The dimensions of a rectangle can include fractional or decimal lengths.

reflection

Writing the Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

Finally, have students fill in the shaded squares in the multiplication table.

reflection

I know to multiply when …

Fill in the shaded squares in the multiplication table below.

×	0	1	2	3	4	5	6	7	8	9	10
0							0	0			
1							6	7			
2							12	14			
3							18	21			
4							24	28			
5							30	35			
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8							48	56			
9							54	63			
10							60	70			

Student pages 13–14

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Multiplication Equations in Context

4

Mathematical Goals

- Understand that multiplication is the operation that is used to join a number of sets (consisting of one thing or collections of things) of identical size

Misconceptions Addressed

- Does not recognize a multiplication situation
- Does not recognize a division situation
- Is unable to transfer between different representations of the same operation

Materials Needed

- ✓ Student pages and markers
- ✓ Response boards
- ✓ Show Me Cards MD4-1 through MD4-6
- ✓ Pen or thin-line marker (two per student in two colors)
- ✓ Connecting cubes or counters
- ✓ Chart paper (for use in setting the direction)

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work.

- amount
- multiplication situation

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards **MD4-1** through **MD4-6**. During today's show me, students will answer questions about multiplication facts involving the number 8. Have students answer the following questions on their response boards.

- **MD4-1** Eight times what number is 64? Show me your answer as an equation.
- **MD4-2** Eight times what number equals 0?
- **MD4-3** How much is 8 times 5?
- **MD4-4** How much is 8 times 1?
- **MD4-5** Seventy-two equals what number times 9?
- **MD4-6** What number equals 8 times 2?

To finish, have students complete the show me problem.

Multiplication Equations in Context 4

show me

Write 8 + 8 + 8 + 8 + 8 as a multiplication equation.

5 × 8 = 40

Student page 17

setting the direction

Introducing the Lesson

Ask a student to read the example problem aloud.

- This is a multiplication problem. What makes this problem a multiplication problem?

  Answer: In order to find out how much ribbon Malaya needs, you have to combine 8 3-foot lengths of ribbon. Multiplication joins a number of sets of equal size. In this case the sets (amounts) are 3-foot lengths of ribbon.

setting the direction

example

Malaya is making bows for 8 packages. She needs 3 feet of ribbon to make each bow.

How much ribbon does she need to make 8 bows?

8 × 3 = 24

Malaya needs 24 feet of ribbon to make 8 bows.

What makes this problem a multiplication story?

There are 8 equal groups. Each group is 3 feet of ribbon. We have to find the total number of feet in 8 groups of 3 feet each.

Student page 17

- Can you think of other types of situations that could be used in multiplication problems?

Brainstorm and chart a list of typical multiplication situations. Leave the list posted.

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- Today, you will be writing multiplication stories to match equations. In your stories, you can use situations such as the ones in this list. Each story should end in a question so that the stories will be the type that we call math stories. After you write each story, make a math drawing or sketch a diagram to match the story, and answer the question that you asked. Remember to show any work that you do!

This lesson offers opportunities for students to engage in constructing viable arguments and critiquing the reasoning of others as they share their work in the probing for understanding discussions. As you observe students working, note:

- Misconceptions (for example, in problem 1, an addition story rather than a multiplication story)
- Partial conceptions (for example, in problem 2, a flawed explanation for why the story is multiplication)
- Work that may move the group's thinking forward (for example, in problem 3, work that treats the missing factor problem as a division problem)

work time

Presenting the Task

Read problem 1 aloud to the group.

scaffolding for success

You may need to walk some students through the task in small steps. First have them identify the number of equal groups and the size of each group, then choose a context that makes sense based on the numbers in the equation, and so on. If students have trouble writing the story in words, encourage them to tell their story with pictures or phrases.

teaching strategies

Watch for students who write stories that use the numbers in the problem but are not multiplication situations. Writing math stories to match a given equation requires a mature understanding of the operation.

work time

1. $2 \times 8 = ?$
   - Write a multiplication story to match this equation.
   - Create a question to go with your story.
   - Make a math drawing or a diagram to match your story.
   - Answer your question using a complete sentence.

Story with Question	
Possible answers: There are 2 rows of apples in a box. Each row has 8 apples. How many apples are there altogether?	
Diagram	**Your Work**
	$2 \times 8 =$ 16
Answer	
There are 16 apples altogether.	

Student page 18

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Reviewing Partner Work

Have students work independently to begin their story.

As students complete their story, they should compare their stories with partners.

probing for understanding

Invite one or two students to share their stories, including the question, a math drawing or diagram, and the answer.

Ask the group questions such as:

- Is this a multiplication story? How do you know?
- How does this math drawing represent the story?
- Do you agree with the answer?

work time

Presenting the Task

Ask a student to read problem 2 aloud to the group.

- Problem 2 asks you to write a story to match a different equation, but in problem 2, you do not have to make a math drawing or diagram and solve the problem. Instead, problem 2 asks you to say how you know your story is a multiplication story.

Students work solo to write their stories, then work with partners to explain why their stories are multiplication stories.

2. $8 \times 10 = ?$

- Write a multiplication story to match this equation.
- Create a question to go with your story.
- Explain how you know your story is a multiplication story.

Story with Question
Possible answers: There are 8 bags with 10 pears in each bag. How many pears are there altogether?
How You Know Your Story Is a Multiplication Story
There are 8 equal groups of pears and you need to find the total number of pears.

Student page 19

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probing for understanding

Invite one or two students to share their stories and explain how they know the story is a multiplication story.

Ask the group questions such as:

- Do you agree with <Name's> explanation?
- Do you have a question for <Name>?

work time

Presenting the Task

Problem 3 is a challenge problem that can be used with students who finish quickly.

3. $8 \times ? = 32$

- Write a multiplication story to match this equation.
- Create a question to go with your story.
- Make a math drawing or a diagram to match your story.
- Answer your question using a complete sentence.

Story with Question	
Possible answers: I have 32 plums and 8 bags. If I put the same number of plums in each of the bags, how many plums will there be in each bag?	
Diagram	**Your Work**
	$8 \times$ 4 $= 32$
Answer	
There will be 4 plums in each bag.	

Student page 20

probing for understanding

Probing should focus on whether the story is a multiplication or a division story. Use this opportunity to focus on the relationship between multiplication and division.

The relationship between these operations is not the main focus of this lesson, so it is not essential that problem 3 be discussed at this time.

teaching strategies

It can be difficult to determine whether a situation is a multiplication or a division situation because students interpret that question differently. For example, some students would see this as a multiplication situation because it would be relatively easy to solve it as an unknown factor problem. Other students would recognize that you divide to solve an unknown factor problem. (If the numbers were greater, this confusion would be less likely to occur.)

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reflection

Writing the Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

Finally, have students fill in the shaded squares in the multiplication table.

reflection

I know that a problem is a multiplication problem when …

Fill in the shaded squares in the multiplication table below.

×	0	1	2	3	4	5	6	7	8	9	10
0							0	0	0		
1							6	7	8		
2							12	14	16		
3							18	21	24		
4							24	28	32		
5							30	35	40		
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9							54	63	72		
10							60	70	80		

Student pages 21–22

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Comparison Situations 5

Mathematical Goals

- Recognizes multiplicative comparison situations with unknowns in different positions
- Understands the difference between additive comparison and multiplicative comparison situations
- Writes equations with a letter standing for the unknown quantity to represent problem situations
- Uses visual representations to represent multiplicative comparison situations
- Uses visual representations to represent additive comparison situations
- Solves one-step word problems

Misconceptions Addressed

- Does not use the correct operation
- Does not recognize addition situations
- Does not recognize subtraction situations
- Does not recognize multiplication situations
- Does not recognize division situations
- Does not write the correct equation to represent a word problem
- Does not know how to represent word problems as diagrams or models

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Scissors and tape (one per student)
✓ Show Me Cards MD5-1 through MD5-5

Optional Materials

✓ Poster paper
✓ Glue (one per student)

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- add
- addition (*adición*)
- difference (*diferencia*)
- subtract
- subtraction (*sustracción*)
- sum (*suma*)
- total

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD5-1 through MD5-5. During today's show me, students will identify which pair of factors is not a factor pair of a number. Have students answer the following questions on their response boards.

- Show me which of these pairs of numbers is *not* a factor pair of …
- … 32 2 and 16 32 and 1 4 and 9 4 and 8
  Answer: 4 and 9
- … 18 2 and 8 3 and 6 9 and 2 18 and 1
  Answer: 2 and 8
- … 30 2 and 15 5 and 6 30 and 1 3 and 9
  Answer: 3 and 9
- … 42 7 and 7 2 and 21 1 and 42 3 and 14
  Answer: 7 and 7
- … 48 6 and 8 2 and 24 15 and 3 4 and 12
  Answer: 15 and 13

To finish, have students complete the show me problem.

Comparison Situations 5

show me

Show me which of these is *not* a factor pair of 24?

24 and 1 2 and 14 4 and 6 8 and 3

Student page 23

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setting the direction

Introducing the Lesson

Read the setting the direction problem to students and have them read along.

Explain this problem involves comparing two quantities—a smaller quantity and a larger quantity—but it involves the difference between the quantities, not how many times greater one is than the other.

setting the direction

Tran has $25. Gabby has $4 more.
How much money does Gabby have?

Estimate: 25 + 5 = 30

Unknown: Let g = amount of money Gabby has.

Diagram:

Equation: $g = 25 + 4$
$g = \$29$

Answer: Gabby has $29.

Student page 23

- What two quantities are being compared?

  Answer: The amounts of money Tran and Gabby have.

- What are the known and the unknown quantities?

  Answer: We know how much money Tran has, $25. This is the smaller quantity. We know that Gabby has $4 more than Tran; this is the difference. We need to find how much money Gabby has, the larger quantity.

- What is a good estimate?

  Answer: Answers will vary. I can round 4 to 5, so a close estimate would be 25 + 5 = 30, or about $30.

- What diagram can you sketch to represent the problem situation?

  Answer: Answers will vary. Students may say tape diagram, number line, or money. Discuss the different ways of representing the problem.

- What equation can you write to represent the problem situation?

  Answer: Sample answer: If we let g = amount of money Gabby has, then $g = 25 + 4$.

- Is the answer reasonable? Explain.

  Answer: Yes, the answer $29 is close to the estimate $30, so $29 is a reasonable answer.

Remind students that all answers should be written in a complete sentence.

scaffolding for success

It may be helpful to color-code the equation with the diagram. For example, draw the tape diagram and the equation on the board. Write the number 25 in red, the number 4 in brown, and the variable, g, in green, in both the diagram and the equation. Students can see the visual relationship between the diagram and equation.

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ERROR

Look for students who have a difficult time understanding additive comparison problems. The difference between the two quantities does not physically exist. It is important that students see visual representations of additive comparison problems so that they can conceptualize this "difference." Tape diagrams provide a good model for representing these problem types. The unknown quantity may be any of the three parts.

work time

Presenting the Task

Read problem 1 instructions aloud as students follow along.

Have students find the *Comparison Situations* page (Student page 25) and cut out all of the cards.

work time

1. a. Get the *Comparison Situations* page (page 25) and cut out all of the cards.
   b. Make four sets of four matching cards. Each set should have one of the following:
   - A word problem card
   - A diagram card
   - An equation card
   - An answer card

   c. When you are done, have your partner check your cards.

Student page 24

Introducing Partner Work

Model partner work for students. Turn to the student closest to you and ask him to be your partner. Using four cards from the lesson, explain to your partner how you know that the cards make a set, and ask him to either challenge the explanation or explain why he agrees. Ask questions if he is unclear in his response.

Model the idea that after partners explain their ideas to each other, they should use the time that remains to modify their work.

Tell students to turn to their partners and share their work, just as you did with your partner.

Observe students as they talk together.

- Is the listener asking questions when the speaker is unclear?
- Do both students explain their processes, even if they did something similar?

COMPARISON SITUATIONS

Mr. Hakim has 12 gold pens. He has 4 times as many red pens as gold pens. How many red pens does Mr. Hakim have?		$12 = 4 + n$ $n = 4 \times 12$
Maria cut off 12 inches of her hair for charity. Anna cut off 4 more inches of hair than Maria. How many inches of hair did Anna cut off?		$n = 12 + 4$ $4 \times n = 12$
Pink dotted shoelaces cost $12. That is 4 times as much as plain white shoelaces cost. How much do the plain white shoelaces cost?		$n = 8$ $n = 3$
Tran buys 12 pounds of oranges and 4 pounds of lemons. How many more pounds of oranges does Tran buy?		$n = 16$ $n = 48$

Student page 25

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Make note of any students who are doing a good job constructing viable arguments or critiquing the reasoning of others so you can highlight these mathematical practices during the probing for understanding discussion.

As you walk around, ask questions such as the following:

- How are you approaching looking for the matches?
- Did you change your mind about any cards in a set? Which ones? How did you identify your error? What did you learn?
- What makes matching the cards difficult? What makes it easy?

scaffolding for success

Card sorting activities encourage students to experiment—they can move the cards around if they change their minds. This type of activity also encourages discussion because students have to explain to each other why they think certain cards match.

Presenting the Task

Have students move on to problem 2 during partner work. Ask students to tell their partners what they are thinking of writing before they write their answers. Instruct them to listen to the their partners' ideas and after they have each had a chance to explain their thinking, write the answer. Remind them to use complete sentences.

If time permits, have partners glue their cards onto poster paper to share in the discussion later in the lesson.

2. Look at your sorted cards.

How do comparison situations involving addition differ from comparison situations involving multiplication?

Possible answer: In addition situations, an amount is added to a quantity, resulting in another quantity. In multiplication situations, a factor is multiplied by a quantity, resulting in another quantity.

Student page 24

scaffolding for success

For English Language learners, the task of writing a "complete sentence" to a math problem can be challenging. When providing examples and/or an explanation of a "complete sentence," be very explicit.

ERROR

Look for students who are having trouble matching problem situations with visuals and equations. Make a note of who may need extra time working on identifying where the known and unknown quantities are in the word problem, in the visual, and in the equation. These students may also need to use manipulatives to model the situation first.

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probing for understanding

Discuss problems 1 and 2 with students. Your role is to facilitate the discussion and to model using correct mathematical language and asking for clarification when an explanation or question is unclear. Ask questions such as the following:

- How did you start the task?
- Which cards were the hardest to match? What made them hard?
- How did you know how to match the diagram to the word problem?
- Where do you "see" the different parts of the equation in the diagram?
- What other equation could you write for this diagram?
- How can estimating help you find the answer?
- What advice would you give someone else who was doing this card sort?

For problem 2, have students explain their answers.

- How can you tell the difference between a comparison situation involving addition and a comparison situation involving multiplication?

  Answer: Answers will vary. Students should state that in an additive comparison situation, two quantities are being compared in order to find their difference. In a multiplicative comparison situation, two quantities are being compared in order to find how many times greater one is than the other.

reflection

Writing a Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

reflection

Something I am still struggling with is …

Student page 27

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Writing Comparison Situations

6

Mathematical Goals

- Recognizes multiplicative comparison situations with unknowns in different positions
- Writes equations with a letter standing for the unknown quantity to represent problem situations
- Writes and solves one-step multiplicative comparison word problems

Misconceptions Addressed

- Does not recognize multiplication situations
- Does not recognize division situations
- Does not write the correct equation to represent a word problem
- Does not use the correct operation

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Chart paper
✓ Show Me Cards MD6-1 through MD6-5

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- mass (*masa*)
- volume (*volumen*)

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD6-1 through MD6-5. During today's show me, students will write an equation for a multiplicative comparison word sentence. Have students answer the following questions on their response boards.

- Show me an equation for this word sentence.
- … 4 times as many as 5 is 20.
  Answer: $4 \times 5 = 20$
- … 54 is 9 times as much as 6.
  Answer: $54 = 9 \times 6$
- … 36 is 6 times as great as n.
  Answer: $36 = 6 \times n$
- … 5 times as many as y is 50.
  Answer: $5 \times y = 50$
- … t times as many as 6 is 42.
  Answer: $t \times 6 = 42$

To finish, have students complete the show me problem.

Writing Comparison Situations 6

show me

Show me an equation for the word phrase given: 15 is 3 times as many as 5.

Possible answers: 15 = 3 × 5 or 15 = 5 × 3

Student page 29

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setting the direction

Introducing the Lesson

Read the instructions and the example problem in setting the direction to students and have them read along.

setting the direction

Gabby wrote the following comparison word problems that can be solved by multiplication or division. She used the three quantities in each problem, with each of the quantities being the answer to one of the three problems.

Quantities	My Problems
4 10 40	Problem: A viper snake in the zoo is 10 inches long. A boa snake is 4 times as long. How long is the boa snake? Unknown: Let s = the length of the boa snake. Equation: $s = 10 \times 4$; $s = 40$ Answer: The boa snake is 40 inches long.
	Problem: Miguel's sunflower is 40 inches tall. This is 4 times as tall as my sunflower. How tall is my sunflower? Unknown: Let h = the height of my sunflower. Equation: $4 \times h = 40$; $h = 40 \div 4$; $h = 10$ Answer: My sunflower is 10 inches tall.
	Problem: A planter box is 10 inches wide and 40 inches long. How many times longer is the planter box than it is wide? Unknown: Let t = the number of times longer. Equation: $t \times 10 = 40$; $t = 40 \div 10$; $t = 4$ Answer: The planter box is 4 times longer than it is wide.

Student page 29

english language learners

These instructions may be difficult for ELL students to understand. Point out the three quantities in the table and how each is used in the three problems. Each quantity is used as the answer to one problem.

Walk through the example problems together. Ask questions such as the following for each one:

- What are the known quantities in the first example problem?

  Answer: The known quantities are the length of the viper snake and how many times as long the boa snake is. So we know the smaller quantity and the multiplier.

- What is the unknown quantity?

  Answer: The length of the boa snake is the unknown. This is the larger quantity.

- Where do you see the known quantities in the equation?

  Answer: In the equation, 4 is the multiplier and 10 is the length of the viper snake.

- Where do you see the unknown in the equation?

  Answer: The unknown is the variable *s*, which represents the length of the boa snake.

scaffolding for success

It may be helpful to write the solution to the example problem on the board, so students have more time to process the information.

teaching strategies

You may need to offer additional support to some students in writing their own word problems. Ask students what they need to think about to write their own word problems. Write what students tell you on chart paper and post it for use as a checklist in this and subsequent lessons. Emphasize that spelling, grammar, and punctuation are not the focus of this lesson.

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work time

Read the instructions for problems 1 and 2 aloud as students follow along.

scaffolding for success

The written problems should involve measures of length, time, volume, mass, or money. Review any unfamiliar terms to be sure students understand what type of problems they should write. You can make a chart of units for each measure and then brainstorm types of problems ahead of time. Students can refer to the chart as they write their problems.

Have students initially work solo on writing the problems. In this lesson, students must look for and make sense of the quantities in the word problems they create. This will be an indication that they are beginning to reason abstractly and quantitatively.

After a few minutes, have students compare their problems with partners to make sure they make sense. Students should revise their problems if necessary. You may ask that certain partners work on particular problems to ensure that all variations are covered.

work time

1. Use the given quantities to write three comparison word problems that can be solved using multiplication or division. Write problems involving length, time, volume, mass, or money.

Use all three quantities in every word problem. Use each quantity as the answer for one of the problems. Possible answers:

Quantities	My Problems
6 9 54	Problem: Maria is 9 years old. Her aunt is 6 times as old. How old is Maria's aunt? Unknown: Let a = the age of Maria's aunt. Equation: a = 9 × 6; a = 54 Answer: Maria's aunt is 54 years old.
	Problem: The brick wall behind our house is 54 inches high. This height is 9 times as high as the border around the garden. How high is the border around the garden? Unknown: Let b = the height of the border. Equation: 9 × b = 54; b = 54 ÷ 9; b = 6 Answer: The border is 6 inches tall.
	Problem: Tran spent $9 at the stationery store. Anna spent $54 at the same store. How many times as much money did Anna spend than Tran at the store? Unknown: Let t = the number of times as much money. Equation: t × 9 = 54; t = 54 ÷ 9; t = 6 Answer: Anna spent 6 times as much money as Tran spent.

2. Write three comparison word problems for the given quantities, as in problem 1. Write problems involving length, time, volume, mass, or money.

Use all three quantities in every word problem. Use each quantity as the answer for one of the problems. Possible answers:

Quantities	My Problems
6 12 72	Problem: A small container of juice holds 12 fluid ounces. A large container of juice holds 6 times as much as a small container. How many fluid ounces of juice does a large container hold? Unknown: Let n = the number of fluid ounces the large container holds. Equation: n = 6 × 12; n = 72 Answer: A large container of juice holds 72 fluid ounces.
	Problem: Mrs. Chi spent $72 on flowers in June. This was 6 times as much money as she spent on flowers in May. How much money did Mrs. Chi spend on flowers in May? Unknown: Let m = the amount of money Mrs. Chi spent on flowers in May. Equation: 6 × m = 72; m = 72 ÷ 6; m = 12 Answer: Mrs. Chi spent $12 on flowers in May.
	Problem: Amir's package weighs 12 ounces. Malaya's package weighs 72 ounces. How many times as much does Malaya's package weigh than Amir's package? Unknown: Let t = the number of times as much Malaya's package weighs. Equation: t × 12 = 72; t = 72 ÷ 12; t = 6 Answer: Malaya's package weighs 6 times as much as Amir's package weighs.

Student pages 30–31

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english language learners

Make sure you demonstrate and explain the activity step by step to ensure that ELL students understand what they are being asked to do. Use partner work time as one way to build in extra supports for ELL students struggling with writing word problems.

- Work with select small groups to scaffold instruction.
- At times, group students so that they can use their native languages to support each other.
- If pairs or groups are not working, try other groupings that may be more successful.

As students are working, walk around the room and look at the problems they have written. Select pairs of students to present their problems in probing for understanding. Be sure to select at least three pairs of students, having each pair present a problem using a different quantity for the unknown.

ERROR

Look for students who write word problems that use the numbers in the problem but are not multiplicative comparison situations. Writing word problems can be difficult and requires a good understanding of the operation. Encourage these students to use manipulatives or sketch diagrams to model the problem situations.

ERROR

Look for students who are unable to write the correct equation to represent a word problem. Help students identify the known and unknown quantities in the problem and in the equations.

probing for understanding

Have pairs present their written problems for problems 1 and 2. For each written problem, ask students the following questions:

- What are the known quantities in your problem?
- What is the unknown quantity?
- How did you define the unknown quantity?
- What equation did you write to represent the situation?
- What is the answer in a complete sentence?
- How does <Name>'s problem compare with <Name>'s problem? What is similar? What is different?

Allow time for students to edit their answers if they need to.

english language learners

This time is a good opportunity for students to work on their English speaking skills and to practice using mathematical language.

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reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished. Have students respond to the reflection prompt.

reflection

Writing comparison situations is (circle one) easy hard for me because …

Student page 32

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Using Data from Graphs with Different Scales

7

Mathematical Goals

- Understands that pictographs that represent large amounts of data work better when they use a scale in which each picture in the graph represents an amount greater than 1
- Understands that when pictures in a pictograph represent amounts greater than 1, it is often necessary to graph part of a picture to represent an amount less than the value of a whole picture
- Understands that bar graphs that represent large amounts of data work better when they use a scale in which each interval along the axis represents an amount greater than 1
- Understands that in bar graphs with intervals greater than 1, it is often necessary to graph a bar to a point between two intervals
- Uses data from scaled pictographs and bar graphs to solve word problems

Misconceptions Addressed

ERROR

- Fails to interpret the scale on a graph or number line
- Thinks that icons on pictographs, boxes on bar graphs, or intervals always represent one of something
- Does not connect the relative lengths of the bars in a bar graph to quantities in the situation

Materials Needed

✓ Student pages and markers

✓ Response boards

✓ Show Me Cards MD7-1 through MD7-5

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD7-1 to MD7-5. During today's show me, students will find the value of a group of symbols based on the value of one symbol. Have students write the answers on their response boards.

- Show me the total value of the symbols.
  - ▲ = 2 ▲▲▲ = ?

    Answer: 6
  - ▭ = 4 ▭▭ = ?

    Answer: 6
  - ● = 3 ●●◐ = ?

    Answer: $7\frac{1}{2}$
  - ⬡ = 2 ⬡⬡⬡⬡◖ = ?

    Answer: 9
  - ♥ = 5 ♥♥♥◖ = ?

    Answer: $17\frac{1}{2}$

To finish, have students complete the show me problem.

Using Data from Graphs with Different Scales
7
show me
If ◯ = 2 CDs, what does ◯◯◯◖ represent?
7 CDs

Student page 33

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work time

Tell students that today they will look at graphs with different types of scales. Explain that on graphs of data that includes large numbers, the scale needs to be adjusted so that the numbers can be represented on a graph.

Have the group read problem 1 aloud together.

Have students work on problem 1 doing solo work to start. Remind them to write their answers in complete sentences. After a few minutes, have students turn to their partners and discuss their answers.

Observe students as they work.

work time

1. This bar graph shows the number of phone calls made to a school during one week in the summer.

a. Describe the scale used for this bar graph.

Possible answer: The scale starts at 0 and increases by 4 each interval.

b. On which day were the fewest calls made to the school? How many calls were made on this day? Explain how you know.

The fewest number of calls were made on Wednesday. There were 6 calls made on Wednesday. The bar is halfway between 4 and 8.

c. On a school day during the fall, the school can get twice as many calls as the number of calls made on Monday and Tuesday combined. How many calls might the school get on a school day in the fall? Explain how you know.

The number of calls on Monday is 22. The number of calls on Tuesday is 28. The total number of calls for Monday and Tuesday combined is 22 + 28, or 50. Twice 50 is 100. The school might get 100 calls on a school day in the fall.

Student pages 33–34

ERROR

Look for students who think that the intervals in the bar graph represent 1, so they misinterpret bars that fall between two intervals. Ask students what each interval represents and what half that number is. Then have students figure out what number is halfway between two intervals.

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probing for understanding

Ask students questions such as these:

- How can you describe the scale for the bar graph in problem 1?

  Answer: The scale starts at 0 and goes to 36. Each interval represents 4 calls.

- Explain how you got your answer in problem 1b.

  Answer: The shortest bar is the bar for Wednesday. The bar extends halfway between 4 and 8, so there were 6 calls on Wednesday.

- How did you solve problem 1c?

  Answer: Answers will vary. I found the number of calls made on Monday and Tuesday and added them to find the total. I then doubled this amount to find the number of calls the school might get in the fall.

- What would this graph look like if it had an interval of 1?

  Answer: The numbers 0 to 36 would be on the vertical axis labeled "Number of Calls." There would be a lot of numbers, which would make the graph difficult to read.

english language learners

To help ELLs with vocabulary, point to the sections of the graph as you discuss them.

scaffolding for success

Partner work is designed to help students consolidate their learning by justifying their solutions to their peers. Students have an opportunity to collaborate with peers as they summarize, analyze, problem solve, explain, and apply new knowledge.

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work time

Tell students that they will look at a pictograph next in which each symbol represents more than 1 thing.

Ask a student to read problem 2 aloud.

Have students work independently on problem 2.

After a few minutes, have students turn to their partner and share their work.

ERROR

Look for students who think each diamond represents 1 letter. Ask these students to look at the legend on the graph and tell you what each symbol represents.

2. This pictograph shows the number of letters the school got during one week in the summer.

Letters Received During the Week

a. Anna said that the school got 6 letters on Friday. Is she correct? Explain how you know.

No, Anna is not correct. Each diamond represents 4 letters. $6 \times 4 = 24$, so the school got 24 letters on Friday.

b. On which day did the school get the most letters? How many letters did the school get on that day? Explain how you know.

The school got the most letters on Tuesday. There are $8\frac{1}{2}$ diamonds. The 8 diamonds are worth 8×4. The half diamond is half of 4, or 2 letters. $(8 \times 4) + 2 = 32 + 2 = 34$, so the school got 34 letters on Tuesday.

c. How many fewer letters did the school get on Thursday than on Monday? Explain how you know.

There are 5 diamonds on Thursday, so the school got 5×4, or 20 letters. There are $6\frac{1}{2}$ diamonds on Monday, so the school got $(6 \times 4) + 2$, or 26 letters. $26 - 20 = 6$. The school got 6 fewer letters on Thursday than on Monday.

Student pages 34–35

probing for understanding

For problem 2, ask questions such as the following:

- Why is Anna wrong?

  Answer: Each diamond represents 4 letters. There are 6 diamonds, so the school got $6 \times 4 = 24$, or 24 letters.

- How did you figure out how many letters the school got on the day it got the most letters?

  Answer: There are the most diamonds on Tuesday, so the school got the most letters on Tuesday. I counted the diamonds. There are $8\frac{1}{2}$ diamonds. Each whole diamond represents 4 letters. Each half diamond represents half of 4, or 2 letters. So, $(8 \times 4) + 2 = 32 + 2 = 34$. The school got 34 letters on Tuesday.

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english language learners

Some ELLs may have difficulty explaining their thinking in English. Allow them plenty of time to formulate their thoughts, and help them with the language if necessary.

- How did you solve problem 2c?

  Answer: Answers will vary. Have students with different solutions share their strategies. Compare the different approaches taken. Some students may find the number of letters received on each day and then find the difference. Other students may find the difference between the numbers of diamonds and then find how many letters that represents.

- What if there were 9 letters received on a given day. How would you graph that on this pictograph?

  Answer: You would have to graph 2 whole diamonds, which would be worth 8 letters. Then you would have to show a quarter of a diamond to represent 1 letter.

- What would this pictograph look like if each diamond represented 1 letter?

  Answer: The pictograph would have to be really large to show all the data.

scaffolding for success

Ask students what questions arose during the lesson. Make sure that all groups of students participate in the discussion.

reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished. Have students respond to the reflection prompt.

reflection

I think bar graphs are easier harder (circle one) for me to read than pictographs because …

Student page 36

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Representing Division

8

Mathematical Goals

- Use math drawings to represent mathematical situations
- Use models to represent mathematical situations
- Use equations and column form to represent division
- Recognize and interpret division situations

Misconceptions Addressed

- Does not recognize a division situation
- Is unable to transfer between different representations of the same operation

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD8-1 through MD8-6
✓ Counters to model the situations, at least 32 per student
✓ Large paper to demonstrate making a math drawing of the situation (11 × 17 inch recommended)

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- divide *(dividir)*
- division *(división)*

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards **MD8-1** through **MD8-6**. During today's show me, students will answer questions involving division by 8. Have students answer the following questions on their response boards.

- **MD8-1** What number equals 48 divided by 8? Show me your answer as an equation.
- **MD8-2** How much is 80 divided by 8?
- **MD8-3** How much is 56 divided by 8?
- **MD8-4** Sixty-four divided by 8 equals what number?
- **MD8-5** What number times 8 equals 32?
- **MD8-6** Twenty-four is equal to 8 times what number?

To finish, have students complete the show me problem.

Representing Division 8

show me

Write all the division facts you can think of for 16.

Possible answers:
16 ÷ 1 = 16
16 ÷ 2 = 8
16 ÷ 4 = 4
16 ÷ 8 = 2
16 ÷ 16 = 1

Student page 37

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work time

Presenting the Task

Read problem 1 aloud to the students and have them read along.

work time

1. A class has 32 students. The teacher wants to split the class into 8 equal teams. How many students will be on each team?

Remember to write your answer as a complete sentence.

32 ÷ 8 = 4

There will be 4 students on each team.

Student page 37

Modeling the Situation with Counters

- I will model this situation using counters.
  - If I had 32 students in our group, I could act this out by actually picking teams. Since there are only <number> of us, I am going to use these counters to represent the students in the class.
  - How many counters would I need to represent the whole class? When I asked myself questions, I said there were 32 students in the class.
  - What does each of these counters represent? Can you help me answer that question?

  Answer: Each counter represents one student.

Remind students that they should always listen to all of the instructions before they begin working with manipulatives. Students should wait for you to say, "Do it now," or "You may begin," which is the signal that you are finished with instructions.

- Let's each take enough counters to represent the students in this story. Do it now.

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Solving the Problem

Tell students to use their counters to show how they would solve the problem. They should pay attention to their steps. Point out the prompt about the first element of a complete solution. After students find the answer to the question, they will write it as part of a complete sentence on the student pages.

scaffolding for success

Depending on the grade level of students, they may not understand what is meant by a complete sentence. You may need to model this for them either now or during probing for understanding.

As students work, observe the ways in which they divide the counters. Watch for students who "know the answer" and just put 4 in each group, students who "deal them out" one at a time or two at a time, and students who make a guess that is too big (so that they run out of counters before they make 8 groups). In the last case, observe whether those students adjust their groups appropriately.

probing for understanding

Ask students who used different or unusual processes to share their methods (or their partners' methods) with the whole group.

If everyone did the same thing, ask:

- Is there any other way we could have divided the counters?

  Answer: If no one has any ideas, "think" of one yourself or make a point of sharing a method you observed while the partners shared. For example, you might put 2 on each "team" first, then count out the rest one at a time, or you might start by putting 6 on each "team" and let the students help you decide how to adjust appropriately.

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work time

Presenting the Task

Tell students that while they read this problem, they should think about what the problem is talking about. What are you counting or measuring? How much or how many are there? What are you trying to find?

Have students work this problem by themselves. Remind them to ask themselves questions and to feel free to use the counters.

Observe students as they begin working on the problem. Do not be too quick to offer additional help. Allow individual struggle time. Offer assurance, not assistance.

2. Anthony has 20 trading cards. He wants to share the cards equally among 5 people.

a. How many cards will each person get?

Possible answer:

20 ÷ 5 = 4

Each person will get 4 cards.

b. How many cards, if any, will be left over?

There will be no cards left over.

Make a math drawing to show how Anthony divided the cards.

Write an equation that you could use to find the answer.

Remember to write your answer as a complete sentence.

Student page 38

teaching strategies

The numbers in this problem are deliberately easy. The real task is for students to learn to show all work, write an equation, make a math drawing or sketch a diagram (if possible), and answer the question using a complete sentence. Later (when the numbers are more difficult) we will also ask for an estimate. Insist that students write an equation, even if they did the computation in their heads or "just knew the answer," because we are trying to lay a path that will eventually lead to algebra.

As students finish, have them compare their responses with partners to make sure they have included all necessary representations and that their work makes sense.

Most students are not used to including so much information in the answer to one problem. Remind them if they forget something, and be consistent in requiring that they include everything that is asked for.

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probing for understanding

Ask questions such as the following to get at students thinking:

- How is this problem like the first one you did?

  Answer: The mathematical situations are identical; the numbers and the context are different.

- Can someone share their math problem in column form?

  Answer: $\begin{array}{r} 4 \\ 5\overline{)20} \end{array}$

- How could we write that as an equation?

  Answer: $20 \div 5 = 4$ or $20/5 = 4$ or $\frac{20}{5} = 4$

  Any one of these is acceptable. It is not necessary that every student write all three forms of the equation.

Allow time for students to begin creating diagrams.

As students work, look for those who make drawings instead of diagrams. Diagrams are "quick and easy." They show the relationships among the quantities and are labeled, but they are not very realistic looking. Note: Do not expect students to label this first diagram.

After about 2–3 minutes, stop students to talk about their work.

Share and ask questions.

Look at several student diagrams and relate them to the situation, the concrete representation, and the symbolic expressions. Ask questions such as:

- What do the dots (or whatever students used) mean?
- How are the people represented in your diagram?
- How is that (point to something in the diagram or one of the numerical expressions) like your counters?
- Which part of the equation goes with the dots?
- Where do we see it in the column form?

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teaching strategies

It is likely that a student who is unable to complete a diagram:

- Is making a drawing with too much detail instead of a diagram, or
- Has no idea what to do

In the first case, work with the student to understand that a diagram is not a realistic picture. Instead, it is a visual representation with labels that illustrates the amounts in the situation and their relationships to each other. In the sample diagrams for problem 2, we see the 20 cards as the total number of dots, the 5 people as enclosed regions or labeled sets, and the 4 cards for each person as the 4 dots in each region or location.

In the second case, it should help when the student sees examples of diagrams that others have made. If no one has any idea what to do, then work together to create a diagram.

reflection

Writing the Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

- Fill in the facts that you know first, then try to use patterns to help you complete the rest of the shaded products.

reflection

My favorite way to do math is (circle one) …

3 + 1 =

because …

Fill in the shaded squares in the multiplication table below.

×	0	1	2	3	4	5	6	7	8	9	10
0							0	0	0		
1							6	7	8		
2							12	14	16		
3							18	21	24		
4							24	28	32		
5							30	35	40		
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9							54	63	72		
10							60	70	80		

Student pages 39–40

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Another Kind of Division

9

Mathematical Goals

- Know that in a division situation the total amount and one other quantity—either the number of groups or the size of each group—are given
- Use models, math drawings or diagrams, equations, and column form to represent mathematical situations

Misconceptions Addressed

ERROR

- Does not recognize a division situation
- Is unable to transfer between different representations of the same operation

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD9-1 through MD9-6
✓ Counters to model the situations (at least 60 per student)

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- compare *(comparar)*
- equal size

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards **MD9-1** through **MD9-6**. During today's show me, students will answer questions about multiplication facts involving the number 9. Have students answer the following questions on their response boards.

- **MD9-1** How much is 9 times 9? Show me your answer as an equation.
- **MD9-2** How much is 9 times 0?
- **MD9-3** What number equals 9 times 2?
- **MD9-4** Thirty-six equals what number times 4?
- **MD9-5** Nine times 1 equals what number?
- **MD9-6** How much is 9 times 10?

To finish, have students complete the show me problem.

Another Kind of Division 9

show me

How many apples? Write an equation that solves this problem.

5 × 9 = 45 or 9 × 5 = 45

Student page 41

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work time

Presenting the Task

Tell students that today they will look at problems where they know the total amount and the size of each group. They will find the number of groups.

Read problem 1 on the student pages together.

Tell students to solve the problem on their own. They can use counters to help them. They should show their work, write an equation, make a math drawing, and write the answer to the question in a complete sentence.

This lesson offers opportunities to highlight the mathematical practice of using appropriate tools strategically as students make math drawings to represent division situations. As you observe students, note any whose work you want to use in probing for understanding in order to highlight this mathematical practice.

As students finish, have them compare their responses with partners to make sure they have included all necessary representations and that their work makes sense.

work time

1. A class has 60 students. The students want to form teams that have 12 students on each team.

a. How many teams can they form?

12 12 12 12 12

24 36 48 60

60 ÷ 12 = ?

The students can form 5 teams.

b. How many students, if any, will be left over?

There will be no students left over.

Use numbers or a math drawing to show how you solved the problem.

Write an equation that you could use to find the answer.

Remember to write your answer as a complete sentence.

Student page 41

teaching strategies

Do not require that students use counters to solve the problem. If a student can move directly to symbols and math drawings, that is acceptable. During a conference, you might want to ask students who do this questions such as: "If you were going to use counters to solve the problem, how many counters would you need? How would you distribute the counters to represent the situation?"

english language learners

Encourage ELL students to use their English speaking skills in class. At the same time allow for use of their native languages to clarify concepts during partner work. New ELL students may benefit from partners who speak the same native language.

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probing for understanding

Ask questions such as:

- What do the dots (or whatever students used) mean?
- How are the students represented in your drawing?
- Where are your counters represented in the drawing/equation/column form problem?
- Which part of the equation goes with the dots (or whatever students used)?
- Where do we see it in the column form?

Allow students to share their representations, especially if someone has recorded the process using a method that has not been seen previously.

ERROR

Notice that today there are 5 groups with 12 in each group. Students who reverse this in their math drawings (12 groups of 5 instead of 5 groups of 12) may not understand the purpose of the drawing or may not see the connection between the drawing and the situation.

scaffolding for success

It is a very common mistake for students to switch the size of each group and the number of groups, especially when the numbers are easy. The answer "pops into their heads" quickly and if there is any uncertainty in their thinking, they become confused.

work time

Presenting the Task

Read the problem together and then have students work on their own.

Have students work independently to solve the problem and record the various representations.

As students finish, have them compare their responses with partners to make sure they have included all necessary representations and that their work makes sense.

Spend some time conferencing with at least one student while partners work together.

2. Tamika has 20 photos. She wants to put the photos into an album so that there are 3 pictures on each page.

a. How many pages will she fill completely?

Possible answer: 1, 2, 3, 4, 5, 6

20 ÷ 3 = 6 R2

Tamika will fill 6 pages.

b. How many photos, if any, will be left over?

There will be 2 photos left over.

Make a math drawing to show how Tamika divided the photos.

Write an equation that you could use to find the answer.

Remember to write your answer as a complete sentence.

Student page 42

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probing for understanding

Ask easier questions.

- What would the answer be if the problem had asked, "How many pages will she need to use if she wants to put all of the photos into the album?"

  Answer: She would use 7 pages instead of filling 6 pages.

- All of the problems that we have done yesterday and today were division problems. What makes a problem a division problem?

  Answer: In a division problem you are separating a total amount into equal-sized groups. You know the total and one other number, either the number of groups or the size of each group.

- What was the same about all of these problems?

  Answer: In each case, we knew the total number of objects (students, teams, photos, trading cards) and one other number.

- Do you think that will be true for all division problems? Explain why.

reflection

Writing the Reflection

When you have about 4 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

Finally, have students fill in the shaded squares in the multiplication table.

reflection

I am confused about …

Fill in the shaded squares in the multiplication table below.

×	0	1	2	3	4	5	6	7	8	9	10
0							0	0	0	0	
1							6	7	8	9	
2							12	14	16	18	
3							18	21	24	27	
4							24	28	32	36	
5							30	35	40	45	
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10							60	70	80	90	

Student pages 42–43

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Inverse Operations 10

Mathematical Goals

- Explore the inverse relationship between multiplication and division and understand that when any number, *a*, is first multiplied then divided by a nonzero number, *n*, the result will be the original number, *a*; understand that the same is also true when any number, a,is first divided and then multiplied by *n*
- Know that for every multiplication equation, $a \cdot b = c$, two matching (or equivalent) division equations can be written—$c \div a = b$ and $c \div b = a$
- Understand that any visual representation that can be used to represent a multiplication situation can be interpreted as an equivalent division situation

Misconceptions Addressed

ERROR

- Fails to link addition and subtraction or multiplication and division as inverse operations
- Does not understand the concept of equivalence

Materials Needed

- ✓ Student pages and markers
- ✓ Response boards
- ✓ Show Me Cards MD10-1 through MD10-6
- ✓ Pen or thin-line marker (two per student in two colors)
- ✓ Scissors (one per student)
- ✓ Glue or tape (one per student pair)

teaching strategies

Before teaching this lesson, prepare a chart to use in modeling the example for problem 1.

Diagram	Equation

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- inverse operation *(operación inversa)*

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards **MD10-1** through **MD10-6**. During today's show me, students will answer questions about multiplication facts involving the number 3. Have students answer the following questions on their response boards.

- **MD10-1** Three times what number equals 21? Show me your answer as an equation.
- **MD10-2** What number equals 3 times 8?
- **MD10-3** What number is equal to 3 times 5?
- **MD10-4** Thirty is equal to 3 times what number?
- **MD10-5** Nine is equal to 3 times what number?
- **MD10-6** How many groups of 2 equal 6?

To finish, have students complete the show me problem.

Inverse Operations 10

show me

Write all the multiplication facts you can think of for the number 36.

1 × 36 = 36	36 × 1 = 36
2 × 18 = 36	18 × 2 = 36
3 × 12 = 36	12 × 3 = 36
4 × 9 = 36	9 × 4 = 36
6 × 6 = 36	

Student page 45

setting the direction

Tell students that today you will be working with inverse operations. Recall that when you know one fact, such as $3 \times 4 = 12$, then you know four facts: $3 \times 4 = 12$, $4 \times 3 = 12$, $12 \div 4 = 3$, and $12 \div 3 = 4$. These four facts are often called a "fact family."

- When you multiply and divide by the same number, you end up with the number that you started with. For example, start with the number 4. $3 \times 4 = 12$; $12 \div 3 = 4$. You are back at 4 again. Operations like multiplication and division that undo each other are called inverse operations.

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work time

Presenting the Task

Ask a student to read the instructions aloud.

Point out to students that they are not being asked to solve math stories, so they do not need to include all of the elements of a complete solution. Students only need to sort the cards and discuss their solution with partners.

Have students begin the task working solo.

Tell students to work with partners to complete work on the card sort.

Spend some time conferencing with students during partner work.

work time

1. Get page 47 and cut out all the cards.
   - Make sets of five cards. Each diagram card matches one multiplication card and its answer card as well as one division card and its answer card.
   - Compare your card sort with your partner's. If they are not the same, try to figure out why.
   - When you both agree that you have sorted the cards correctly, tape or glue them to your poster.

Example

• • • • • •	3 × 2	6	6 ÷ 2	3

Diagram Cards

Multiplication Cards	Division Cards	Answer Cards	
8 × 5	40 ÷ 5	9	35
6 × 9	35 ÷ 7	14	40
5 × 7	54 ÷ 6	5	112
8 × 14	112 ÷ 8	54	8

Student pages 45 and 47

probing for understanding

Ask students:

- Did you and your partner agree on which cards to place together?

Call on a student (if there is one) who answered "no" and ask her to tell where she and her partner disagreed. Have both partners explain their reasoning about where to place the figure in question. Then, ask the whole group to help them resolve the disagreement.

- How did you decide which cards to group together?
- Which cards were the hardest to match? What made them hard?
- How do you "see" the multiplication expression in the diagram?
- How do you "see" the division expression in the diagram?
- Why can we use the same diagram for both multiplication and division?

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reflection

Writing the Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

Finally, have students fill in the shaded squares in the multiplication table.

reflection

Something that is still hard for me is …

Fill in the shaded squares in the multiplication table below.

×	0	1	2	3	4	5	6	7	8	9	10
0				0	0		0	0	0	0	0
1				3	4		6	7	8	9	10
2				6	8		12	14	16	18	20
3	0	3	6	9	12		18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5					20		30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Student pages 46 and 49

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Using Multiplication to Solve Division Problems

11

Mathematical Goals

- Understand equivalent equations using multiplication and division
- Use multiplication to check division
- Interpret math stories

Misconceptions Addressed

- Fails to link addition and subtraction or multiplication and division as inverse operations

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD11-1 through MD11-6
✓ Sheet of paper with the equations $7 \times 13 = 91$ and $91 \div 13 = x$ written on it large enough for students to see

Optional Materials

✓ A large copy of the example problem to use in the initial demonstration

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- equivalent *(equivalente)*
- inverse operation
- quotient

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards **MD11-1** through **MD11-6**. During today's show me, students will answer questions involving division by 5. Have students answer the following questions on their response boards.

- **MD11-1** Six times what number equals 30? Show me your answer as an equation.
- **MD11-2** Five times what number equals 10?
- **MD11-3** Fifty is equal to 5 times what number?
- **MD11-4** Zero divided by 5 equals what number?
- **MD11-5** Twenty-five is equal to what number times 5?
- **MD11-6** What number times 7 is equal to 35?

To finish, have students complete the show me problem.

Using Multiplication to Solve Division Problems 11

show me

Write 45 – 5 – 5 – 5 – 5 – 5 – 5 – 5 – 5 – 5 = 0 as a division equation.

45 ÷ 9 = 5

Student page 51

setting the direction

Tell students they will work on inverse operations.

Ask students to think back to the show me they just did. Multiplication and division go together; they are inverse operations.

- Who remembers what multiplication and division being inverse operations means?

  Answer: If you multiply by a number then divide the product by the same number, you will be back where you started.

scaffolding for success

It may be helpful to write the word "inverse" on the board and then give some real-world examples of inverse, such as putting a glove on and taking it off, pulling something apart and putting it back together, and so on. This will give students a more concrete representation of the concept.

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- So, if you know that $7 \times 13 = 91$, then you also know the answer to the problem $91 \div 13 = x$. Who can tell me what the quotient is without actually dividing?

english language learners

If time permits, you may want to have students make a concept map for the word "inverse."

Tell students that it is often easier to multiply than to divide. Knowing the inverse relationship can help students especially when they take multiple-choice tests. Students can also use multiplication to check their answer after they divide.

Ask students to read the problem silently.

Analyze the problem together.

- Suppose you were given this problem to solve without the multiple-choice answers. How would you go about solving it? Take a minute and talk to your partner to decide how you would solve it.

  Answer: This problem is a division situation. You know the total amount ($66) and the size of each group ($3).

setting the direction

example

Miguel saves $3 each week. How many weeks will it take him to save $66?

Ⓐ 18 weeks Ⓑ 2 weeks

Ⓒ 22 weeks Ⓓ 3 weeks

I can rule out 2 weeks and 3 weeks because they are too small.

I know $3 \times 20 = 60$ and $3 \times 2 = 6$,

so $3 \times 22 = 66$.

It will take 22 weeks for Miguel to save $66.

Student page 51

- Since this problem is a multiple-choice question, we can actually find the answer without dividing. The first thing we always do is make an estimate. Talk to your partner again, and make an estimate of the number of weeks.

  Answer: One example of a reasonable estimate is that $3 × 20 is $60, so the number of weeks should be a little greater than 20.

- With a multiple-choice question, some of the answers are usually unreasonable. Can you eliminate any of the answer choices based on your estimate?

  Answer: Even without an estimate, students should recognize, using what they know about the magnitude of the number 3 and the multiplication facts 3×2 and 3×3, that answers Ⓑ and Ⓓ can be eliminated. Depending on how students estimated, they may also be able to eliminate answer Ⓐ.

- Now you are down to one or two possibilities. Instead of dividing, check by multiplying.

Have different sets of partners check the remaining answer or answers (to speed up the process). Students will find that only one of the possibilities, 22 weeks, gives an answer of $66.

Have students mark their student pages appropriately.

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work time

Presenting the Task

Ask a student to read problem 1 aloud to the group.

english language learners

An alternative to having an individual student read the problem aloud is to have the group read it aloud. Reading aloud in this way gives ELLs practice with the language, without being singled out.

work time

1. Tran needs to buy 75 hot dog buns. There are 8 buns in a package.

   How many packages should he buy?

   (A) 10 packages (B) 9 packages
   (C) 8 packages (D) 80 packages

First estimate to rule out wrong answers.

Then multiply to find the correct answer.

I eliminated 80 because it is too big.

I know 8 × __ = 75 or 8 × ? = 75.

I tried 9 and got 8 × 9 = 72.
That is not enough, so I know the answer is 10 packages.

Student page 52

Observe students as they begin working on the problem.

As students finish, have them compare their responses with those of their partners. During partner work, you should conference with students as needed.

scaffolding for success

If students are struggling, suggest they use a number line.

probing for understanding

Allow students to share their answers.

- How did you get your estimate?

  Answer: One possible method would be to multiply 10 by 8. So, 10 packages would be 80 hot dog buns, which is a little greater than 75.

- Are any of the answers unreasonable?

  Answer: Students should be able to eliminate answer (D).

- Based on your estimate, are you able to eliminate any other answers?

  Answer: Answers (A), (B), and (C) are so close to each other and so close to the likely estimate of 10 packages that students may reasonably feel they cannot eliminate any of those answers. Some students may think they can eliminate (A) because 10 × 8 is greater than 75; without considering the problem situation, 80 appears to be "too much" or "too high."

- How could you use multiplication to find the answer?

  Answer: Using recall of multiplication facts, students may try 9 × 8 first because 9 times is closer to 75 than 10 times, the likely estimate.

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- What else do you have to think about to find the answer?

  Answer: Besides getting the mathematical answer, you have to think about the real-world situation. Because the hot dog buns come in packages, there might not be a number of packages that gives you the exact number of hot dog buns you need. Because it will not work to have fewer than 75 hot dog buns, you have to buy a number of packages that has at least 75 hot dog buns.

work time

Presenting the Task

Read problem 2 aloud. Be sure everyone knows what a quotient is.

Observe students as they begin working on the problem.

As students finish, have them compare their responses with partners. During partner work, you should spend some time conferencing as needed.

2. Anna thinks the answer is B. Anna is wrong.

   How can she use multiplication to see her mistake?

   8 × 9 = 72. Tran will be short 3 buns if he buys 9 packages.

   Tran needs 10 packages to get at least 75 hot dog buns.

Student page 52

probing for understanding

Ask questions like the ones you asked in probing for understanding for problem 1.

- If you were solving this problem, which answer choices could you eliminate because they are obviously wrong? Why do you think they are wrong?

  Answer: The answer 80 packages should be eliminated because that would be greater than a package for each hot dog bun needed.

- What might Anna have been thinking that caused her to choose B?

  Answer: This questions presents an opportunity to talk about interpreting the quotient or the product (depending on how you were solving the problem.) Students often ignore remainders instead of reasoning about them. Watch for students who make "real-world" assumptions like "There will probably be a couple of people who do not want hot dog buns, so 72 is enough." Discuss with these students the fact that if you make an assumption you have to include that assumption in your response. Otherwise, the response will be considered wrong.

- Why might another student choose B? What common mistake might that student be making?

  Answer: If you use multiplication to solve the problem and realize that you have to buy 80 hot dog buns, you might choose 80 as your answer.

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reflection

Writing the Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

Finally, have students fill in the shaded squares in the multiplication table.

reflection

I still have questions about …

Fill in the shaded squares in the multiplication table below.

×	0	1	2	3	4	5	6	7	8	9	10
0				0	0	0	0	0	0	0	0
1				3	4	5	6	7	8	9	10
2				6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Student pages 53–54

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Connecting Representations in a Problem

12

Mathematical Goals

- Solve multiplication and division problems in context

Misconceptions Addressed

- Is unable to transfer between different representations of the same operation

Materials Needed

✓ Student pages and markers

✓ Response boards

✓ Show Me Cards MD12-1 through MD12-6

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards **MD12-1** through **MD12-6**. During today's show me, students will answer questions about multiplication and division facts involving the number 2. Have students answer the following questions on their response boards.

- **MD12-1** Two groups of 6 is equal to what number? Show me your answer as an equation.
- **MD12-2** What number times 2 equals 0?
- **MD12-3** Four equals what number times 2?
- **MD12-4** Two is equal to what number divided by 1?
- **MD12-5** How much is 10 divided by 2?
- **MD12-6** What number equals 18 divided by 2?

To finish, have students complete the show me problem.

Connecting Representations in a Problem 12

show me

Solve for n.

$7 \times n = 42$ n = 6

Student page 55

setting the direction

Tells students they will work with math stories.

Presenting the Task

Read the example problem aloud to the students. Walk through the problem together. Be sure that students note the solution recorded on the student pages to use as an example as they work on the rest of the problems.

setting the direction

example

Fill in this table.
All of the parts should go with the problem.

PROBLEM
3 children are going to share 72 plums equally.
How many plums does each child get?

ESTIMATE	EQUATION
3 × 10 = 30 3 × 20 = 60 More than 20	72 ÷ 3 = n
DRAWING	**CALCULATION**
	3 × 20 = 60 3 × 4 = 12 3 × 24 = 60 + 12 = 72

ANSWER
Each child gets 24 plums.

Student page 55

english language learners

Modeling is an easy and effective strategy to use when working with ELL students. Demonstrating an activity step-by-step allows students to focus on the task rather than the language embedded in the instructions.

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scaffolding for success

It may be helpful to write your solution to the example problem on the board, so that students have more time to process the information.

When you represent the situation visually, be sure to show 3 groups of 24, not 24 groups of 3. Also, be sure to mention that equations can consist of numbers, or numbers and unknown amounts (letters or symbols to represent the unknown amount). This fact will be important when the problem includes a remainder. Writing an equation that includes a remainder is difficult unless you write the remainder as a fraction or a decimal.

Ask questions such as the following:

- How did you know that this is a division situation?

  Answer: We know the total and the number of equal groups. We are trying to find the size of each group.

- Where do you see the total in the representation?

  Answer: The total is represented by all the dots or the end of the number line. (Answers can vary depending on the representation.)

- Where do you see the 3 groups?

  Answer: In the 3 circles or the 3 arrows

- Where do you see the number in each group?

  Answer: The number of dots in a circle or the length of an arrow

- Where do you see the total in the equation (column form arithmetic)?

  Answer: The total is the dividend.

- Where do you see the 3 groups?

  Answer: The group of 3 is the divisor.

- Where do you see the number in each group?

  Answer: The number in each group is the quotient.

scaffolding for success

Be sure to point to the different parts of the solution as you talk about them.

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work time

Presenting the Task

Ask a student to read problem 1 aloud.

Explain to students that in the rest of the problems, they will complete whatever parts of the table are missing. Sometimes the table includes a math story. Other times, it might already have an equation or a visual representation, but the math story is missing. When the math story is missing, part of the task is for students to come up with a math story that matches the equation or the representation. In each table, all of the parts need to match—they all need to fit the same mathematical situation.

Observe students as they begin working. Focus your help on making sure they understand what they are to do, rather than on how to do it. Allow individual struggle time. Offer assurance, not assistance.

work time

1. Fill in this table.
   All of the parts should go with the problem.

   Check your work with your partner as you go along.

PROBLEM	
Maria is wrapping packages. She has 90 feet of ribbon to make bows. She needs 6 feet of ribbon for each bow. How many bows can she make?	
ESTIMATE Possible estimates: 100 ÷ 5 = 20 100 ÷ 10 = 10	EQUATION Possible equations include: 90 ÷ 6 = n 90 ÷ 6 = 15 6 × ? = 90 6 × 15 = 90
DRAWING 30 30 30	CALCULATION 5 × 6 = 30 5 × 6 = 30 5 × 6 = 30 15 × 6 = 90
ANSWER Maria can make 15 bows.	

Student page 56

After a few minutes, have students share their work on problem 1 with their partners. Students should revise their work if necessary.

As students are working, walk around and look at the representations and equations that they have written. Select students to present their work in probing for understanding. Be sure to select a variety of representations and equations, if possible.

Spend some time conferencing with a student during partner work, as needed.

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probing for understanding

This problem is very similar to the example problem. Hopefully, students will not have a great deal of difficulty with it. This problem differs from the example problem in two ways:

- It involves continuous measures.
- We know the total and the size of each equal group and we are trying to find how many groups (bows) we can make.

Ask questions such as the following:

- In this problem, are we counting or measuring?

  Answer: Both. We are measuring ribbon, but we are counting the number of 6-foot lengths we can make.

- What amounts are mentioned in this problem?

  Answer: The amounts mentioned are 90 feet of ribbon (starting amount), 6 feet of ribbon per bow, and the number of bows that can be made.

- What amounts have numbers that are given in the problem?

  Answer: The total amount of ribbon that Maria has (90 feet) and the amount of ribbon needed for each bow (6 feet)

- Where do you see those amounts in your drawing?
- Where do you see those amounts in your equation?
- Where do you see those amounts in your calculations?
- What amount are we looking for?

  Answer: The number of bows

- Where do you see that amount in your drawing? In your equation? In your calculations?

Watch for students who represent this problem incorrectly as 6 groups of 15 instead of 15 groups of 6. If this incorrect representation occurs, ask students to look at two different representations—one that shows 6 groups of 15 and one that shows 15 groups of 6.

- Where do you see the 6 feet of ribbon for each bow in each of these drawings?
- Which of these drawings more accurately shows this problem situation?

A drawing that shows 6 groups of 15 does not make sense. Separating the 90 feet of ribbon into 6 discrete parts does not mirror the situation; that would mean that you have 6 bows with 15 feet of ribbon in each bow.

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work time

Presenting the Task

Tell students that this problem is similar to the two that we have already done, but it gives you an equation to start with instead of a math story.

Read the directions for problem 2 aloud to the students.

Make sure everyone knows what to do, and let students get to work.

english language learners

Some students may have trouble with the task of writing a math story. Pair these students with students who can help them with the language during partner work.

2. This time you are given the equation. Make up a problem that goes with it. Fill in the rest of the table. All of the parts should go with the problem you make up.

Check your work with your partner as you go along.

PROBLEM	
Answers will vary.	
ESTIMATE	EQUATION
Answers will vary.	$13 = 91 \div 7$
DRAWING	CALCULATION
Answers will vary.	Answers will vary.
ANSWER	
Answers will vary.	

Student page 57

Observe students as they begin working. Focus your help on making sure they understand what has been assigned, not on how to do it. Allow individual struggle time. Offer assurance, not assistance.

After a few minutes, have students compare their problems with partners to make sure their problems make sense. Students should revise their work if necessary.

As students are working, walk around and look at the math stories, drawings, and equations that they have written. Begin selecting students to present their work.

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probing for understanding

As time allows, have the students you have selected share their work on problem 2.

Encourage the group to ask the presenter for clarification if the explanation of any part is unclear or they need to hear it again in order to understand (for example, a calculation method may be new to some students, who may ask to have it explained again).

Ask questions similar to these:

- Who has a question for <Name>?
- Was any part of <Name's> explanation unclear to you? What would you like to ask about?

reflection

Writing the Reflection

When you have about 3 minutes left, stop the discussion, even if you are not finished. Have students respond to the reflection prompt.

Finally, have students fill in as much of the multiplication table as they can.

- Before you begin the reflection, let's look at the multiplication table together. Today's multiplication table is completely blank! I want you to fill in as much of the table as you can as a review. Fill in the facts that you know first, then try to use patterns to help you complete the rest of the table.

reflection

Writing math stories is ☺ easy for me ☹ hard for me. (circle one)

because …

Fill in the shaded squares in the multiplication table below.

×	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	68	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Student pages 58–59

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Multiplication and Division Word Problems

13

Mathematical Goals

- Recognizes multiplication situations involving equal groups with unknowns in different positions
- Recognizes that division can be used to solve an unknown-factor problem
- Writes equations with a letter standing for the unknown quantity to represent problem situations
- Solves one-step multiplication and division word problems

Misconceptions Addressed

ERROR

- Does not recognize multiplication situations
- Does not recognize division situations
- Thinks a variable represents an object rather than a number
- Does not use the correct operation
- Fails to link multiplication and division as inverse operations

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD13-1 through MD13-5
✓ Chart paper

Optional Materials

✓ Base-10 blocks
(large collection of flats, longs, and unit cubes)

✓ Counters
(large collection for student use as needed)

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- divide *(dividir)*
- dividend *(dividendo)*
- division *(división)*
- divisor *(divisor)*
- equal groups
- factor *(factor)*
- multiply *(multiplicar)*
- multiplication *(multiplicación)*
- product *(producto)*
- quotient *(cociente)*

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD13-1 through MD13-5. During today's show me, students will focus on the relationship between multiplication and division by finding all related multiplication and division equations for a set of three numbers.

- Show me all the multiplication and division equations you can write using …
  - … 7, 8, and 56.

    Answer: $7 \times 8 = 56$, $8 \times 7 = 56$, $56 \div 8 = 7$, $56 \div 7 = 8$, $56 = 7 \times 8$, $56 = 8 \times 7$, $7 = 56 \div 8$, $8 = 56 \div 7$
  - … 27, 9, and 3.

    Answer: $3 \times 9 = 27$, $9 \times 3 = 27$, $27 \div 9 = 3$, $27 \div 3 = 9$, $27 = 3 \times 9$, $27 = 9 \times 3$, $3 = 27 \div 9$, $9 = 27 \div 3$
  - … 6, 36, and 6.

    Answer: $6 \times 6 = 36$, $36 \div 6 = 6$, $36 = 6 \times 6$, $6 = 36 \div 6$
  - … 7, 28, and 4.

    Answer: $7 \times 4 = 28$, $4 \times 7 = 28$, $28 \div 4 = 7$, $28 \div 7 = 4$, $28 = 7 \times 4$, $28 = 4 \times 7$, $7 = 28 \div 4$, $4 = 28 \div 7$
  - … 72, 8, and 9.

    Answer: $9 \times 8 = 72$, $8 \times 9 = 72$, $72 \div 8 = 9$, $72 \div 9 = 8$, $72 = 9 \times 8$, $72 = 8 \times 9$, $9 = 72 \div 8$, $8 = 72 \div 9$

To finish, have students complete the show me problem.

Multiplication and Division Word Problems 13

show me

Show me all the multiplication and division equations you can write using 5, 6, and 30.

5 × 6 = 30, 6 × 5 = 30, 30 ÷ 5 = 6, 30 ÷ 6 = 5

30 = 5 × 6, 30 = 6 × 5, 6 = 30 ÷ 5, 5 = 30 ÷ 6

Student page 61

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setting the direction

In today's lesson, students will work with equal-groups multiplication and division situations. The following table shows three common situations.[1] It is important that students are asked to represent and solve for all unknowns.

setting the direction

Mr. Hakim asked Anna to write three word problems. He said that Anna should use all three quantities in each word problem and each quantity should be used as the answer for one problem. Anna wrote the following problems:

example

Quantities	My Problems
6 7 42	PROBLEM: There are 7 packages of pens. There are 42 pens in all. How many pens are in each package? UNKNOWN: Let p = the number of pens in each package. EQUATION: 7 × p = 42 or p = 42 ÷ 7 p = 6 ANSWER: There are 6 pens in each package.

example

	My Problems
	PROBLEM: There are 42 pens in all. There are 6 pens in a package. How many packages are there? UNKNOWN: Let p = the number of packages. EQUATION: p × 6 = 42 or p = 42 ÷ 6 p = 7 ANSWER: There are 7 packages.
	PROBLEM: There are 6 pens in a package. There are 7 packages of pens. How many pens are there in all? UNKNOWN: Let p = the total number of pens. EQUATION: p = 7 × 6 p = 42 ANSWER: There are 42 pens in all.

Student pages 61–62

	Unknown Product	Group Size Unknown	Number of Groups Unknown
Equal Groups of Objects	There are 3 bags with 2 plums in each bag. How many plums are there in all? 3 x 2 = ?	If 6 plums are shared equally in 3 bags, then how many plums are in each bag? 3 × ? = 6 and 6 ÷ 3 = ?	If 6 plums are packed 2 to a bag, then how many bags are needed? ? × 2 = 6 and 6 ÷ 2 = ?

[1]Adapted from Box 2–4 of *Mathematics Learning in Early Childhood*, National Research Council (2009, pp. 32, 33).

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Assess students' prior knowledge of multiplication and division situations involving equal groups. Sketch one of the following diagrams (use the diagram that makes the most sense to you) on chart paper to review the three quantities in an equal-groups situation. Show how each part relates to a multiplication equation and a division equation. Model a simple situation with counters, such as 2 groups of 3 counters each for a total of 6 counters: $2 \times 3 = 6$, $6 \div 3 = 2$, and $6 \div 2 = 3$. Keep the diagram posted for students to refer to as they work.

Total quantity to be separated into equal-sized groups				
Size of Each Group	Size of Each Group	...	Size of Each Group	Size of Each Group

or

Total quantity to be separated into equal-sized groups	
Number of Groups	Size of Each Group

$a \times b = c$

$c \div a = b$ and $c \div b = a$

english language learners

In some countries, the equation $2 \times 3 = 6$ means 3 groups of 2 things each for a total of 6 things instead of 2 groups of 3 things each for a total of 6 things. Discuss the different interpretations and let ELLs use the one that is most comfortable for them.

Read the example problem in setting the direction as students follow along.

Explain that two of the quantities will be in a word problem and the third quantity will be the answer (the unknown quantity). Point out that each problem is missing a different one of the three quantities. On a board or chart paper, draw a diagram of the three quantities. Point to each quantity as you discuss each example problem.

Total number of pens	
Number of Packages	Number of Pens in Each Package

- What are the known quantities in the first problem of the example?

  Answer: The known quantities are the number of packages of pens, 7 packages, and the total number of pens, 42 pens.

- What is the unknown quantity in the first problem of the example?

  Answer: The unknown quantity is the number of pens in each package.

- If p represents the unknown quantity, what equation can you write to represent the situation?

  Answer: $7 \times p = 42$ or $p = 42 \div 7$

- What does p equal?

  Answer: $p = 6$

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- What is the answer in a complete sentence?
  Answer: There are 6 pens in each package.

Continue in a similar manner with the second and third problems in the example.

english language learners

Make sure you demonstrate and orally explain the activity step by step to ensure that ELLs understand what they are being asked to do.

work time

Read the directions for problems 1 and 2 aloud as students follow along.

Have students work solo on writing the problems. In this lesson, students look for and make sense of the quantities in the word problems they create. Students should demonstrate that they are beginning to make sense of quantities and their relationships in problem solving.

Introducing Partner Work

Model partner work for students. Turn to the student closest to you and ask him to be your partner. Using a problem from today's work time, explain to your partner how you got your answer, and ask him to tell you what he did. Ask questions if he is unclear in his explanation.

Model the idea that after partners read or explain their ideas to each other, they should use the time that remains to edit their work in order to make it clearer.

Tell students to turn to their partners and share their work, just as you did with your partner.

- Observe students as they talk together.
- Is the listener asking questions when the speaker is unclear?
- Do both students explain their processes, even if they did something similar?

As students complete their word problems, they should turn and talk to their partner. Have students compare their problems with a partner to make sure they make sense. Students should revise their problems if necessary. You may ask that certain partners work on particular problems to ensure that all variations are covered.

work time

Use the given quantities to write related word problems. All the quantities should be used in every word problem. Each quantity should be used as the answer for one problem.

Possible answers:

	Quantities	My Problems
1.	4 9 inches 36 inches	PROBLEM: Anthony has 36 inches of ribbon. He cuts the ribbon into pieces that are each 9 inches long. How many pieces of ribbon does Anthony have now? UNKNOWN: Let n = the number of pieces of ribbon Anthony has now. EQUATION: n × 9 = 36 or n = 36 ÷ 9 n = 4 ANSWER: Anthony has 4 pieces of ribbon now.
		PROBLEM: Anthony has some ribbon. He cuts the ribbon into 4 pieces that are each 9 inches long. How long was Anthony's ribbon to start with? UNKNOWN: Let n = the number of inches long the ribbon is to start. EQUATION: n = 4 × 9 n = 36 ANSWER: Anthony's ribbon is 36 inches long to start.
		PROBLEM: Anthony has 36 inches of ribbon. He cuts the ribbon into 4 pieces of equal length. How long is each piece? UNKNOWN: Let n = the number of inches long each piece is. EQUATION: 4 × n = 36 or n = 36 ÷ 4 n = 9 ANSWER: Each piece is 9 inches long.

Student pages 63–64

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english language learners

Use partner work time as one way to build in extra supports for English language learners who are struggling with writing word problems.

- Work with select small groups to scaffold instruction.
- At times, group students so that they can use their home language to support each other.
- If pairs or groups are not working well together, try other groupings that may be more successful.

As you observe students working, look at the problems they have written. Select pairs of students to present their problems in probing for understanding. Be sure to select at least three pairs of students, having each pair present a problem using a different quantity for the answer.

ERROR

Look for students who write word problems that use the numbers in the problem but are not multiplication or division situations. Writing related word problems can be difficult and requires a good understanding of the operation. Encourage these students to use manipulatives to model the problem situations.

ERROR

Look for students who are unable to write an equation using a variable for the unknown quantity or who misunderstand what the variable represents. Students should realize that the variable is an unknown quantity and does not represent an object, such as ribbon. It is the *number* of pieces of ribbon or the *length* of the ribbon, not the ribbon itself. Be sure students include number or length in the description of the unknown.

Possible answers:

2.

Quantities	My Problems
6 $8 $48	PROBLEM: Tran buys 6 pairs of socks that cost $8 each. How much money do the socks cost in all? UNKNOWN: Let n = the amount of money the socks cost in all. EQUATION: n = 6 × 8 n = 48 ANSWER: The socks cost $48 in all.
	PROBLEM: Tran spends $48 on some socks. Each pair costs $8. How many pairs of socks does Tran buy? UNKNOWN: Let n = the number of socks Tran buys. EQUATION: n × 8 = 48 or n = 48 ÷ 8 n = 6 ANSWER: Tran buys 6 pairs of socks.
	PROBLEM: Tran buys 6 pairs of socks. The socks cost $48 in all. Each pair of socks costs the same amount. How much money does one pair of socks cost? UNKNOWN: Let n = the amount of money one pair of socks costs. EQUATION: 6 × n = 48 or n = 48 ÷ 6 n = 8 ANSWER: Each pair of socks costs $8.

Student pages 64–65

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english language learners

ELLs will probably not be familiar with this use of the word *given*. Ask students what they think *given* means in the first sentence of the directions. Have a brief discussion about what the word refers to and point out where the given quantities appear in the chart. Also discuss the term *related*. Talk about how brothers and sisters are related because they are in the same family. These problems will be related because the equations used to solve the problems will be in the same "fact family."

probing for understanding

Have pairs present their word problems from problems 1 and 2. For each written problem, ask students the following questions:

- What are the known quantities in your problem?
- What is the unknown quantity?
- How did you define the unknown quantity?
- What equation did you write to represent the situation?
- What is the answer in a complete sentence?

english language learners

This is a good opportunity for ELLs to work on their speaking skills and to practice using mathematical language.

teaching strategies

Make a table for problems 1 and 2, such as the ones shown, and have students point to the known quantities and the unknown quantity as they talk about their word problems.

Problem 1

Total Length of the Ribbon in Inches	
the number of pieces of ribbon	the length of each piece of ribbon in inches

Problem 2

Total Cost of the Socks	
the number of pairs of socks Tran buys	the cost of one pair of socks

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work time

Read problem 3 aloud.

> 3. How can you find an unknown factor if you know one factor and the product?
> You can use division to find the unknown factor.

Student page 65

Have students work independently and then share their answer with their partner.

ERROR

Look for students who do not recognize the relationship between multiplication and division as inverse operations. Encourage students to find where the product is in a related division equation (dividend) and where the factors are (quotient and divisor).

probing for understanding

Use this time to focus on the relationship between multiplication and division. Have students share their methods for finding an unknown factor if one factor and the product are known.

- How can you find an unknown factor if you know one factor and the product?

  Answer: You can use division because they are opposite operations. For example, if I know $2 \times 6 = 12$, then I also know $12 \div 6 = 2$ and $12 \div 2 = 6$.

Allow time for students to edit their answers if they need to.

teaching strategies

It can be difficult to determine whether a situation is a multiplication or a division situation, because different people may interpret that question differently. For example, some students would see one problem as an unknown-factor problem, while other students would recognize that you divide to solve an unknown-factor problem. With numbers that are greater, this confusion is less likely to occur.

reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished. Have students respond to the reflection prompt.

> **reflection**
>
> It was easy hard (circle one) for me to write the related word problems because …

Student page 66

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More Multiplication and Division Word Problems

14

Mathematical Goals

- Recognizes multiplication situations involving arrays and area with unknowns in different positions
- Writes equations with a letter standing for the unknown quantity to represent problem situations
- Uses visual representations to represent multiplication situations involving arrays and area
- Solves one-step multiplication and division word problems

Misconceptions Addressed

ERROR

- Does not recognize multiplication situations
- Does not recognize division situations
- Does not know how to represent word problems as diagrams or models
- Does not use the correct operation

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD14-1 through MD14-5

Optional Materials

✓ Square tiles (28 tiles for demonstration purposes)

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. A Spanish cognate is in parentheses.

- array
- area *(área)*
- row

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD14-1 through MD14-5. During today's show me, students will use their understanding of division as an unknown-factor problem to find the missing factor in a multiplication equation. Have students answer the following questions on their response boards.

- Show me …
  - … what number n equals. What division equation can you use to solve it?
    $n \cdot 5 = 45$

    Answer: $n = 9$; $45 \div 5 = 9$
  - … what number x equals. What division equation can you use to solve it?
    $6x = 42$

    Answer: $x = 7$; $42 \div 6 = 7$
  - … what number c equals. What division equation can you use to solve it?
    $21 = c \times 3$

    Answer: $c = 7$; $21 \div 3 = 7$
  - … what number y equals. What division equation can you use to solve it?
    $2 \cdot y = 18$

    Answer: $y = 9$; $18 \div 2 = 9$
  - … what number b equals. What division equation can you use to solve it?
    $49 = 7b$

    Answer: $b = 7$; $49 \div 7 = 7$

To finish, have students complete the show me problem.

More Multiplication and Division Word Problems 14

show me

Show me what number d equals. What division equation can you use to solve it?

$32 = 8 \times d$ d = 4, 32 ÷ 8 = 4

Student page 67

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setting the direction

In today's lesson, students will work with multiplication and division situations involving arrays and area. The table below shows six common situations.[1] It is important that students are asked to represent and solve for all unknowns.

Read the setting the direction instructions aloud to students. Tell students that today they will play the role of teacher and assess Maria's completed homework. They should review Maria's work and correct the answer if it is wrong. Then they should explain what Maria did wrong.

setting the direction

Maria turned in her math homework.

a. Mark whether each problem is right or wrong.

b. Correct the problems she got wrong.

c. With your partner, discuss what Maria did wrong and write an explanation.

Name: Maria

Mrs. Chi arranges 28 chairs into 4 equal rows for a class poetry reading. How many chairs are in each row?

4

28

c = the number of chairs

c = 40 + 40 + 32

c = 112

There are 112 chairs.

Maria's answer is wrong. Maria thought that each row had 28 chairs and she was trying to find how many chairs total.

4 rows

c = number of chairs in each row

There are 28 chairs in all.

4 × c = 28 or c = 28 ÷ 4

c = 7

There are 7 chairs in each row.

Student page 68

	Unknown Product	Unknown Factor	Unknown Factor
Arrays,[2] Area[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? Area example: What is the area of a 3 centimeter by 6 centimeter rectangle?	If 18 apples are arranged in 3 equal rows, how many apples are in each row? Area example: A rectangle has area 18 square centimeters. If one side is 3 centimeters long, how long is a side next to it?	If 18 apples are arranged in equal rows of 6 apples, how many rows are there? Area example: A rectangle has area 18 square centimeters. If one side is 6 centimeters long, how long is a side next to it?

[1] Adapted from Box 2–4 of *Mathematics Learning in Early Childhood*, National Research Council (2009, pp. 32, 33).

[2] The language in the array examples shows the easiest form of array problems. A harder form uses the terms *rows* and *columns*: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

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Ask a student to read the problem Maria is solving in setting the direction.

english language learners

Keep in mind that some students may not feel comfortable reading aloud. Be prepared to support them in this task.

Ask the following questions:

- What are the known quantities?

  Answer: The total number of chairs and the number of equal rows

- What is the unknown quantity?

  Answer: The number of chairs in each row

- What does Maria's sketch show?

  Answer: 28 chairs in each of 4 rows

- Does Maria's sketch match the problem situation? Explain.

  Answer: No, Maria should have sketched a total of 28 chairs arranged equally in 4 rows.

Have a student come to the board to sketch the correct arrangement or use square tiles to show it.

- Where is the unknown quantity in this sketch?

  Answer: The unknown quantity is the number of chairs in a row.

english language learners

Introduce the term *array* to describe an arrangement of objects organized into rows and columns, with the same number of objects in each row. Create a poster titled Array and show examples of arrays as a visual reminder of its definition. Include rectangular arrays with squares touching without any gaps or overlaps to represent area. Label these examples Area.

Continue working through the correct solution with students. Have students record on their student pages as you discuss each part of the solution.

- If you let c equal the unknown quantity, what does c represent?

  Answer: c represents the number of chairs in a row.

- Can c represent a chair?

  Answer: c must represent a quantity. It can represent 1 chair, but not the chair itself.

- What equation could you use to represent the problem situation?

  Answer: $4 \times c = 28$ or $c = 28 \div 4$

- What is the answer in a complete sentence?

  Answer: There are 7 chairs in each row.

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work time

Have a student read the instructions for problems 1 and 2 aloud.

Ask students to work independently on problems 1 and 2. Remind students of the elements of a complete solution.

english language learners

Provide students with manipulatives (square tiles, for example), so that they can explore, make connections, and build conceptual understanding.

After a few minutes, have students share with their partners what they think Maria did wrong. Remind students to be good listeners and ask questions when they are not clear about their partner's explanations. This lesson offers opportunities for students to engage in constructing viable arguments and critiquing the reasoning of others as they share their work.

work time

Maria turned in her math homework.

- Mark whether each problem is right or wrong.
- Correct the problems she got wrong.
- With your partner, discuss what Maria did wrong and write an explanation.

Name: Maria

1. A lawn is shaped like a rectangle. The lawn has an area of 56 square yards. If one side of the lawn is 8 yards long, how long is a side next to it?

y = length of the side in yards

y = 56 + 8; y = 64; The other side is 64 yards long.

Maria's answer is wrong. She thought one side of the lawn was 56 yards, and the other side was 8 yards longer. The area of the lawn is 56 square yards. One side is 8 yards long. We are trying to find the length of the side next to it.

y × 8 = 56 or y = 56 ÷ 8; y = 7

The length of the side next to it is 7 yards.

Name: Maria

2. For band practice, 50 students are arranged into equal rows of 10 students. How many rows of students are there?

n = the number of students in the other rows

50 students in all

10 + n = 50 or n = 50 − 10; n = 40

There are 40 students in the other rows.

Maria is wrong. She did not understand the problem.

She found the number of students in the other rows, not the number of rows of students.

r = the number of rows

r × 10 = 50 or r = 50 ÷ 10; r = 5

There are 5 rows of students.

Student pages 69–70

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english language learners

Having students work in pairs provides them with opportunities to negotiate meaning, share ideas, and communicate with others during problem solving. This type of language-rich environment in which students are encouraged to work together and learn from one another is essential for ELLs' learning.

ERROR

Look for students who do not understand multiplication situations involving arrays. The "language of rows" can be difficult. Help students see that an array situation can be thought of as an equal-groups situation if each row is considered an equal group. Then students can determine the number of equal groups (rows) or the number of objects in a group (number in each row).

probing for understanding

Ask the following questions for problems 1 and 2. Try to have a different set of partners answer each problem.

- In problem 1, what are the two known quantities?
  Answer: The area of the lawn and the length of one side
- What is the unknown quantity?
  Answer: The length of the side next to the side with the known length
- Does Maria's sketch represent the problem situation accurately?
  Answer: No, Maria thought one side of the lawn was 56 yards and the other side was 8 yards longer.

Have partners come to the board and sketch a diagram to match the problem situation.

- Where is the unknown quantity in your sketch?
  Answer: The length of the side next to the side with the known length

teaching strategies

Use tiles to show that the area of a rectangle involves arrays of squares with no gaps or overlaps, so the area of a rectangle is a special type of array problem situation. The shortcut for finding the area of a rectangle is to multiply the length (the number of objects in a row) by the width (the number of rows). Remind students that area is given in square units.

- If y equals the unknown quantity, what equation can you write to match your sketch?
  Answer: $y \times 8 = 56$ or $y = 56 \div 8$
- What is the answer in a complete sentence?
  Answer: The length of the unknown side is 7 yards.

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- In problem 2, what are the known quantities?
  Answer: The total number of students and the number of students in each row
- What is the unknown quantity?
  Answer: The number of rows of students
- Does Maria's sketch represent the problem situation accurately?
  Answer: No, Maria found the number of students in the other rows, not the number of rows.

Have the pair of students sketch a diagram to match the problem situation.

- Where is the unknown quantity in your sketch?
  Answer: The unknown quantity is the number of rows.
- If r equals the unknown quantity, what equation can you write to match your sketch?
  Answer: $r \times 10 = 50$ or $r = 50 \div 10$
- What is the answer in a complete sentence?
  Answer: There are 5 rows of students.

work time

Read problem 3 aloud as students read along.

Have students work independently and then discuss their answer with their partner.

3. How do you know when a problem involves multiplication or division?

Possible answer:

Multiplication involves counting groups of equal size to find how many in all or finding the area of a rectangle when given the length and width. Division is just finding an unknown factor.

Student page 71

english language learners

For this task, encourage ELLs to explain their ideas to one another. Math language must be used. Encourage the use of English without discouraging students from using their first language(s).

reflection

Writing a Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

reflection

To help Maria solve these problems, I would tell her …

Student page 71

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One-Step Word Problems 15

Mathematical Goals

- Recognizes addition situations
- Recognizes subtraction situations
- Recognizes multiplication situations
- Recognizes division situations
- Writes equations with a letter standing for the unknown quantity to represent problem situations
- Uses visual representations to represent problem situations
- Solves one-step word problems
- Uses mental computation and estimation strategies to check the reasonableness of answers

Misconceptions Addressed

ERROR

- Does not recognize addition situations
- Does not recognize subtraction situations
- Does not recognize multiplication situations
- Does not recognize division situations
- Does not know how to represent word problems as diagrams or models
- Does not use the correct operation
- Estimates incorrectly

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD15-1 through MD15-5
✓ Scissors and tape (one per student)

Optional Materials

✓ Base-10 blocks
(large collection of flats, longs, and unit cubes)
✓ Square tiles
(large collection for student use as needed)
✓ Poster paper (one per student pair)
✓ Glue (one per student)

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD15-1 through MD15-5. During today's show me, students will use their understanding of subtraction as an unknown-addend problem to find the missing addend in an addition equation. Have students answer the following questions on their response boards.

scaffolding for success

Be sure to offer students a variety of ways to access the mathematics, both visually and contextually. Special needs students benefit greatly from tasks such as card sorts, in which they match different representations of the same mathematical information. Representations might include equations, diagrams, descriptions in words, and number lines. In pairs or groups, students can also create posters that summarize mathematical concepts and terminology and require visual organization of concepts.

- Show me …
  - … what number b equals. What subtraction equation can you use to solve it? $b + 34 = 69$

  Answer: $b = 35$; $69 - 34 = 35$
  - … what number c equals. What subtraction equation can you use to solve it? $105 = 72 + c$

  Answer: $c = 33$; $105 - 72 = 33$
  - … what number y equals. What subtraction equation can you use to solve it? $77 + y = 124$

  Answer: $y = 47$; $124 - 77 = 47$
  - … what number n equals. What subtraction equation can you use to solve it? $205 = n + 100$

  Answer: $n = 105$; $205 - 100 = 105$
  - … what number f equals. What subtraction equation can you use to solve it? $f + 32 = 91$

  Answer: $f = 59$; $91 - 32 = 59$

To finish, have students complete the show me problem.

One-Step Word Problems 15

show me

Show me what m equals. What subtraction equation can you use to solve the problem?

$37 = m + 19$

m = 18; 37 − 19 = m

Student page 73

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work time

Tell students that in this lesson, they will be asked to match equations, answers, and visual representations with one-step word problems.

Read the instructions for problem 1 aloud as students follow along.

Have each student find the *Word Problem, Diagram, Equation, and Answer Cards* page (student page 75) and cut out all the cards.

Ask students to sort the cards into sets of four, with each set containing a word problem card, a matching diagram card, an equation card, and an answer card.

work time

1. Work with your partner.
   - Get the *Word Problem, Diagram, Equation, and Answer Cards* page (page 75) and cut out all the cards.
   - Make four sets of four matching cards. Each set should have one of the following:
     - word problem card
     - diagram card
     - equation card
     - answer card
   - Explain to your partner how you know that the cards match. Your partner should either agree with your explanation or challenge it if your explanation is not clear, correct, and complete.
   - Once you and your partner agree on the matches, tape the cards together in this order: word problem, diagram, equation, and then answer.

   After you tape your cards, work on the problems that follow.

Student page 73

Once students think they have sorted the cards correctly, they should check with their partners to make sure. Students should explain how they know that the cards match. Their partner should either agree with the explanation or challenge it if the explanation was not clear, correct, or complete.

After both partners agree with their card sorts, check their matches. If the cards are correctly grouped, have students tape the cards in each set together in a row. Make sure that students tape the cards in the correct order.

Look carefully at each student's card sort as they are working. If you see errors or misconceptions in student work today, plan to work with students in an extended lesson or during a conference to correct their work.

WORD PROBLEM, DIAGRAM, EQUATION, AND ANSWER CARDS

Malaya covers a rectangular table top with square tiles. She uses 56 tiles. There are 8 tiles in each row. How many rows of tiles are there?	$n = 8$		$n = 56 \div 7$
	Mr. Samson has 56 inches of rope. He wants to cut it into 7 equal pieces. How long will each piece of rope be?	$n \times 8 = 56$	$n = 48$
$n = 49$		Amir baked 56 muffins for a bake sale. At the end, there were 8 muffins left. How many muffins did Amir sell at the bake sale?	$7 + n = 56$
$8 = 56 - n$	Gabby has \$7. She earns some money feeding the neighbor's pets. She now has \$56. How much money did Gabby earn feeding the neighbor's pets?	$n = 7$	

Student page 75

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Card Sort Solution

Malaya covers a rectangular table top with square tiles. She uses 56 tiles. There are 8 tiles in each row. How many rows of tiles are there?		$n \times 8 = 56$	$n = 7$
Mr. Samson has 56 inches of rope. He wants to cut it into 7 equal pieces. How long will each piece of rope be?		$n = 56 \div 7$	$n = 48$
Amir baked 56 muffins for a bake sale. At the end, there were 8 muffins left. How many muffins did Amir sell at the bake sale?		$8 = 56 - n$	$n = 48$
Gabby has $7. She earns some money feeding the neighbor's pets. She now has $56. How much money did Gabby earn feeding the neighbor's pets?		$7 + n = 56$	$n = 49$

As students are working, ask questions such as these:

- How are you approaching looking for the matches?
- Did you change your mind about any cards in a set? Which ones? How did you identify your error? What did you learn?
- What makes matching the cards difficult? What makes it easy?

scaffolding for success

You may need to guide some students through the task in small steps. First have them cut out all of the cards, then start with one word problem and solve it, then look for the related diagram card, and so on.

ERROR

Look for students who are still having trouble matching problem situations with visuals and equations. Make a note of who may need extra time working on identifying where the known and unknown quantities are in the word problem, in the visual, and in the equation. These students may also need to use manipulatives to model the situation first.

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scaffolding for success

Note misconceptions and errors that emerge. Make sure students:

- Take their time without rushing through the task
- Take turns, so everyone participates
- Explain their reasoning
- Ask questions when their partner is unclear
- Verify and challenge each other's reasoning text

Have students move on to problems 2 and 3 during partner work. Ask students to tell their partner what they are thinking of writing before they write their answer. Instruct them to listen to their partner's ideas and, after they have both had a chance to explain their thinking, write the answer on their student pages.

If time permits, have students glue their card sorts onto poster paper to share in the discussion later in the lesson.

2. What other equation can you write for each word problem? Possible answers:

a. Malaya's problem $n = 56 \div 8$

b. Mr. Samson's problem $7 \times n = 56$

c. Amir's problem $56 = 8 + n$

d. Gabby's problem $n = 56 - 7$

3. Estimate the answer to Gabby's problem. Explain why the given answer is reasonable.

Possible answer:

$57 - 7 = 50$

I think Gabby earned about $50. $50 is close to the answer $49, so the answer is reasonable.

Student page 74

ERROR

Look for students who simply guess at an estimate instead of using a strategy. It is important to have students articulate their reasoning. Help these students see the difference between a guess and an estimate. Ask questions such as, "Will the answer be greater or less than 57? How do you know?"

scaffolding for success

Partner work provides students with the opportunity to work cooperatively in order to negotiate and share ideas and communicate with each other during problem solving. Such group-learning settings provide special needs students a chance to hear and use mathematical language while building their communication skills.

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probing for understanding

Discuss problems 1 and 2 with students. Your role is to facilitate the discussion and to model using correct mathematical language and asking for clarification when an explanation or question is unclear. Ask questions like the following:

- How did you start the task?
- Which cards were the hardest to match? What made them hard to match?
- How did you know how to match the diagram to the word problem?
- Where do you "see" the different parts of the equation in the diagram?
- What other equation could you write for this diagram?
- How can estimating help you find the answer?
- What advice would you give someone else who was doing this card sort?

For problem 3, have students explain their estimation strategy.

- How did you get your estimate?

  Answer: Answers will vary. It is easy to subtract 7 from 57, so I rounded 56 to 57. Then I got 57 – 7 = 50. My estimate is about $50.

- Did anyone estimate a different way? Explain.

  Answer: Answers will vary. If no one had a different method, ask students to think of another way. For example, round 57 to 60 and 7 to 10. Then 60 – 10 = 50, so about $50. Or, round 57 to 55 and 7 to 5 because it is easier to calculate 55 – 5 = 50. The estimate is also about $50.

teaching strategies

Having students listen to other students' estimation strategies will help them develop strong estimation skills. Students can and will learn strategies from each other. They will also learn that there is not one "right" strategy and that many estimates are possible.

reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished. Have students respond to the reflection prompt.

reflection

I still have questions about …

Student page 77

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Hidden Questions and First-Step Equations

16

Mathematical Goals

- Understands that in two-step problems, there is one unknown quantity that must be solved and then used to find another unknown that answers the problem question. Identifies what that unknown quantity, or "hidden question," is in a two-step problem.
- Writes equations with a letter standing for the unknown quantity to represent problem situations.

Misconceptions Addressed

ERROR

- Does not recognize the implied data in a word problem—unnamed quantities that must be calculated in multistep problems

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD16-1 through MD16-5
✓ Scissors and tape (one per student)

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD16-1 to MD16-5. During today's show me, students will write an equation they can use to solve a one-step word problem. Have students answer the following questions on their response boards.

- Show me an equation you can write to solve the word problem. Use n to represent the unknown quantity.
  - Mrs. Chi buys some snack bars. There are 54 snack bars without nuts. There are 18 snack bars with nuts. How many snack bars does Mrs. Chi buy?

    Answer: $n = 54 + 18$

  - If 64 cans of tuna are arranged into 8 equal rows, how many cans of tuna are in each row?

    Answer: Equations may vary. Sample equations: $8 \times n = 64$; $n = 64 \div 8$

  - Josh picks 24 fewer oranges than Anthony. Anthony picks 53 oranges. How many oranges does Josh pick?

    Answer: Equations may vary. Sample equations: $n = 53 - 24$; $n + 24 = 53$

  - If 42 shirts are to be packed 6 to a box, then how many boxes are needed?

    Answer: Equations may vary. Sample equations: $n \times 6 = 42$; $n = 42 \div 6$

  - The flower bed at the park measures 8 feet long and 3 feet wide. What is the area of the flower bed?

    Answer: $n = 8 \times 3$

To finish, have students complete the show me problem.

Hidden Questions and First-Step Equations 16

show me

There are 33 balls on the tennis court. 19 balls are near the net. The rest are near the back fence. Show me an equation you can write to find how many balls are near the back fence.

Possible answers:

$19 + b = 33$, $b = 33 - 19$

Student page 79

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work time

In this lesson, students will be given some two-step word problems, hidden question cards, and first step cards. Working in pairs, students must match the cards to the related word problem. The first step cards are equations that students can use to answer the hidden question. Students are not asked to find the solution to the problems during this lesson.

work time

Work with a partner and follow these instructions for problems 1 – 4.

- Get the *Hidden Question Cards* and *First Step Cards* pages (pages 83 and 85) and cut out all of the cards.
- Match each hidden question card to a word problem on pages 80 and 81. There will be four cards that you will not use.
- Match each first step card to a word problem. The first step cards say what equation you might do first. There will be four cards that you will not use.
- Explain to your partner how you know that the cards match. Your partner should either agree with your explanation or challenge it if your explanation is not clear, correct, and complete.
- Once you and your partner agree on the matches, tape the cards on pages 80 and 81.
- Do not solve any problems yet.

Student page 79

scaffolding for success

Card-sorting activities encourage students to experiment—they can move the cards around if they change their minds. This type of activity also encourages discussion, because students have to explain to each other why they think certain cards match.

Read the instructions for problems 1–4 aloud as students follow along. Make sure students understand that they will be considering all four problems at the same time.

Have students work in pairs on this activity.

Have each student find the *Hidden Question Cards* and the *First Step Cards* pages (student pages 83 and 85) and cut out all the cards.

Ask pairs to read each word problem on student pages 80 and 81 and to match a hidden question card and a first step card with a word problem. Explain that the equations on the first step cards can be used to answer the hidden question.

Once pairs think they have matched the cards correctly, they should check with another pair to make sure.

After students agree with their card sorts, ask them to invite you to check them. If the cards are correctly grouped, have students tape the cards beneath the related word problem. Then, have students describe how they matched the cards to the word problems.

When students are finished with the sorting activity have them answer problem 5.

english language learners

Encourage students to work cooperatively. For this problem, you may want to group students who are fluent in English with students who are not fluent so that they can assist each other with the problems.

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english language learners

Modeling is an easy and effective strategy to use when working with ELLs. Demonstrating an activity step by step allows students to focus on the task rather than on the language embedded in the instructions.

As students are working, spend some time conferencing with one or more students individually. Recognizing the reasonableness of intermediate results is part of the mathematical practice of looking for regularity in reasoning. The ability to translate the hidden question into a mathematical expression fits that criterion.

5. Describe how you matched the cards to the problems.

Possible answer:
The equations on the first step cards are used to solve the related hidden questions. We first identified the hidden question and then used the question to find the equation.

Student page 82

ERROR

Look for students who are still having trouble identifying the hidden question in a two-step problem. Ask students to identify the quantities needed to solve the problem. Then have students underline in the word problem where the quantities can be found. One will be an unknown quantity. Ask students how they can find the unknown quantity and identify this question as the hidden question.

HIDDEN QUESTION CARDS

How many fluid ounces of paint does Tamika buy in total?	How wide is Mrs. Chi's garden plot?
How much more blue paint than gold paint does Tamika buy?	How long is Mr. Samson's garden plot?
How much more money does Amir have in his pocket than Tran did to start?	How much shorter is Gabby's piece of plastic lace than Miguel's piece?
How much money does Tran have in his pocket after finding 4 cents?	How long is Miguel's piece of plastic lace?

Student page 83

FIRST STEP CARDS

$n = 32 + 8$	$32 + 8 = n$
$n = 32 - 8$	$32 + 4 = n$
$n = 32 + 4$	$n + 8 = 32$
$n = 8 + 4$	$n \times 8 = 32$

Student page 85

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Card Sort Solution

1. Tamika buys 32 fluid ounces of blue paint and 4 fluid ounces of gold paint. Anna buys 8 fewer fluid ounces of paint than Tamika. How many fluid ounces of paint does Anna buy?

How many fluid ounces of paint does Tamika buy in total?	$n = 32 + 4$

2. Tran has some coins in his pocket. He finds 4 cents and adds it to his pocket. Amir now has 8 cents more than Tran. Amir has 32 cents in his pocket. How much money did Tran have in his pocket to start with?

How much money does Tran have in his pocket after finding 4 cents?	$n + 8 = 32$

3. Mrs. Chi's garden plot at school is 32 square feet. The length of the plot is 8 feet. Mr. Samson's garden plot is the same length as Mrs. Chi's, but it is 4 feet wider. How many feet wide is Mr. Samson's garden plot?

How wide is Mrs. Chi's garden plot?	$n \times 8 = 32$

4. Gabby has a piece of plastic lace that is 32 inches long. Miguel has a piece of lace that is 8 inches longer than Gabby's. Miguel cuts his piece of lace into 4 equal pieces. How long is each of Miguel's pieces?

How long is Miguel's piece of plastic lace?	$n = 32 + 8$

Student pages 80–81

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probing for understanding

Discuss the card sort for problems 1–4. Ask questions like the following:

- How did you complete the card sort?
- Why do you think that hidden question matches that word problem?
- Why do you think that first step card equation matches that word problem?
- What are the known and unknown quantities? Where do you see them in your equation?
- What advice would you give someone else who was doing this card sort?

Then ask students to share their answers to problem 5.

reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished. Have students respond to the reflection prompt.

reflection

I still have questions about …

Student page 82

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Multiplying by Multiples of 10

17

Mathematical Goals

- Understands the concept of place value for whole numbers and that a digit in one place represents 10 times what it represents in the place to its right
- Multiplies one-digit whole numbers by multiples of 10, using place value and properties of operations in the range 10–90 (e.g., 9×80, 5×60) and using strategies based on place value and properties of operations
- Identifies arithmetic patterns (including patterns in the addition table or multiplication table), and explains them using properties of operations

Misconceptions Addressed

ERROR

- Does not recognize or misapplies the commutative property
- Does not recognize or misapplies the distributive property
- Does not recognize or misapplies the commutative property
- Does not recognize or misapplies the distributive property

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD17-1 through MD17-5

Optional Materials

✓ Chart paper
✓ Base-10 blocks (10 tens blocks and 10 unit blocks)

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- associative property
- commutative property
- distributive property
- factor *(factor)*
- multiple *(múltiplo)*
- multiplication table *(tabla de multiplicación)*
- pattern *(patrón)*
- relationship *(relación)*

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD17-1 through MD17-5. During today's show me, students will demonstrate their ability to think quickly when comparing two products and their understanding of the commutative property of multiplication. Have students answer the following questions on their response boards.

- Show me the symbol (greater than or less than) that makes the statement true.
  - 4×4 ____ 6×3

    Answer: <
  - 6×7 ____ 5×8

    Answer: >
  - 7×8 ____ 6×9

    Answer: >
- Show me a multiplication equation for each array.
  -

    Answer: $4 \times 2 = 8$, $2 \times 4 = 8$
  -

    Answer: $4 \times 7 = 28$, $7 \times 4 = 28$

Make a note of students who have difficulty with the commutative property so that you can address their difficulties with them in a later lesson, either informally during partner work or by conferencing.

To finish, have students complete the show me problems.

Multiplying by Multiples of 10 17

show me

Show me the number that completes this equation.

$5 \times 3 =$ 3 $\times 5$

Draw a diagram to show that your equation makes sense.

Answers will vary. Correct drawings are likely to show arrays or sets.

or

Student page 87

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setting the direction

teaching strategies

The information included in setting the direction is extensive. Be sure to read through this information before this lesson. You may not be able to cover everything that is suggested. Decide in advance how you will prioritize the discussion and be prepared to cut the discussion short, if necessary, to allow time for the rest of the lesson.

Read the setting the direction instructions as students follow along. Ask students to think about 7 × 2 × ____ = 14 × 10 = 140.

- 7 times 2 times what number equals 14 times 10?

  Answer: 10

- How do you know that 10 is correct?

  Answer: The first factor, 7, is in the equation, so the second factor, 20, must be represented in the equation by the "2 times something." That has to be 2 × 10. Multiplying by 2 and then by 10 is the same as multiplying by 20.

- The associative property means that numbers can be multiplied in any order to get the same total. How does this equation 7 × 2 × 10 = 14 × 10 show the associative property?

  Answer: When the problem is written as 7 × 2 × 10, you can multiply any two of the numbers first. In this problem, 7 × 2 is done first, to make 14, and then 14 is multiplied by 10.

setting the direction

Write 7 × 20 in different ways:

7 × 2 × 10 = 14 × 10 = 140	Associative Property
7 × 5 × 4 = 35 × 4 = 140	Associative Property
7 (10 + 10) = 70 + 70 = 140	Distributive Property
7 (5 + 5 + 5 + 5) = 35 + 35 + 35 + 35 = 140	Distributive Property

Student page 87

english language learners

Allow students to use their first language when exploring and explaining the terms for properties of operations. By allowing them to use their first language, you are lowering their "affective filters." By lowering their affective filters, you promote a safe learning environment.

Ask students to think about 7 × 5 × ____ = 35 × ____ = 140.

- What numbers did you use to fill in the blanks?

  Answer: 4 is used in both blanks.

- How is this problem like the first problem we looked at?

  Answer: This problem also uses the associative property. In this instance, 20 is broken into 5 times 4. The 7 and 5 are multiplied first, and that product is multiplied by 4.

- Why does writing 4 in both blanks make the equation correct?

  Answer: Since 7 × 5 is equal to 35, for the two multiplication statements to be equivalent, the remaining factors must be the same.

Ask students to think about 7(10 + 10) = 70 + 70 = 140.

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- How did you rewrite 7(10 + 10)?

  Answer: 70 + 70.

- How do you know that 7(10 + 10) = 70 + 70?

  Answer: The product, 140, is shown. Multiplying each 10 in the parentheses by 7 gives 70 + 70, and that adds up to 140.

- The distributive property means that these two ways are the same: multiplying the sum of two numbers, and multiplying each of the two numbers and then adding the results. How does this equation, 7(10 + 10) = 70 + 70, show the distributive property?

  Answer: This equation, 7(10 + 10) = 70 + 70, shows that multiplying the sum of 10 + 10 (which is 20) by 7 is equivalent to multiplying one 10 by 7, multiplying the other 10 by 7, and then adding up the results, 70 and 70.

teaching strategies

Do not expect students to know the formal definition of the distributive property—for any quantity $(b + c)$, $a(b + c)$ is the same as $a(b) + a(c)$—or the other properties they use in this lesson. Instead, encourage students to develop meaningful and precise descriptions of the properties in their own words.

Ask students to think about 7(5 + 5 + 5 + 5) = ____ + ____ + ____ + ____ = 140.

- How is this problem like the last one, 7(10 + 10)?

  Answer: This problem also uses the distributive property. Here, 20 is broken into 5s instead of into 10s. Then the 5s are multiplied by the other factor, 7; in the last problem, the 10s were multiplied by 7.

scaffolding for success

The equations include two types of notation for multiplication: the multiplication sign ×, and the juxtaposition of quantities surrounded by parentheses, as in 7(10 + 10). Make sure students understand both notations.

teaching strategies

If students claim to know these properties, do not simply assume that they understand them. The distributive property is fundamental to many of the algorithms that students use in arithmetic and algebra. Students will use the associative and distributive properties in the work time activity. Plan to follow the progress of students who are unfamiliar with or who are having difficulty with the properties now.

Keep the focus on exploring how to partition factors. The goal is for students to be secure in the idea that numbers can be broken apart in different ways to make easier partial products and to be comfortable with ways of doing so.

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work time

Read problem 1 aloud as students follow along. Ask students to turn to page 14 of their student pages.

Make sure all students are familiar with the multiplication table for basic multiplication facts and understand how the table on their student pages is to be completed.

Tell students that they should start to fill in the table individually.

teaching strategies

Do not try to correct students at this stage. The idea is for students to learn to identify, describe, and correct their own errors.

work time

1. Fill in this multiplication table.

×	10	20	30	40	50	60	70	80	90
1	10	20	30	40	50	60	70	80	90
2	20	40	60	80	100	120	140	160	180
3	30	60	90	120	150	180	210	240	270
4	40	80	120	160	200	240	280	320	360
5	50	100	150	200	250	300	350	400	450
6	60	120	180	240	300	360	420	480	540
7	70	140	210	280	350	420	490	560	630
8	80	160	240	320	400	480	560	640	720
9	90	180	270	360	450	540	630	720	810

Student page 88

Observe whether students seem to be working in a systematic way. You might ask questions to learn about their approaches and understanding, such as, "How do you know that $3 \times 70 = 210$?" Students with strong base-ten understanding think of 70 as 7 tens. Multiplying 3×70, then, is the same as thinking about 3 groups of 7 ten or 21 tens, which is 210. This conceptual understanding is the basis for the calculation pattern that students are likely to use to complete the table: find the product of the non-zero digits and attach a zero to shift the product one place to the left, making the result 10 times as large.

ERROR

Watch for students who recognize a pattern, or apply a procedure, without strong conceptual understanding. If students are writing the times facts and attaching a zero to each product, check for understanding. Note students you want to pay special attention to during probing for understanding.

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ERROR

Students who seem unclear on the effects of multiplying by 10 may benefit from modeling using base-10 blocks. Display one unit block. Ask students to show 10 times that much with blocks. Next, have students write the numbers shown (1 and 10). Ask, "What can you say about what happens to the ones block when it is multiplied by 10?" Repeat this process using other numbers of unit blocks until students can clearly articulate that ones become tens when multiplied by 10 and that as the value of the digit changes, so does its place.

After a few minutes of solo work, have students check in with their partner to complete the multiplication table.

Have students move on to problem 2 for more partner work. Remind them to be good listeners and ask questions when they are not clear about their partners' explanations.

Encourage students to communicate their thinking clearly. Tell them to come to agreement with their partners on the description of each pattern or relationship.

Possible patterns might include:

- The first row is the same as the first column, the second row is the same as the second column, and so on because of the commutative property of multiplication—the order in which two numbers are multiplied does not affect the product.
- The sum of each row or column is 450 more than the previous one because of the distributive property. Any number times (10 + 20 + 30 + 40 + 50 + 60 + 70 + 80 + 90) is the same as that number times 450.
- The 10s and 20s columns add up to the 30s column because of the distributive property. Any number times (20 + 10) is the same as that number times 30. For example, 4(20 + 10) = 4 × 30.

2. With your partner, look for at least two patterns in the multiplication table. Then explain the patterns using the properties of operations.

example

The products are the same as the times facts but ten times as large.

You can write the times fact and attach a zero. The times fact moves to the left, making each digit ten times as large.

Possible answers:

The first row is the same as the first column, the second row is the same as the second column, and so on. This is true because of the commutative property of multiplication—the order in which two numbers are multiplied does not affect the product.

The sum of each row or column is 450 more than the previous one. This is true because of the distributive property. Any number times (10 + 20 + 30 + 40 + 50 + 60 + 70 + 80 + 90) is the same as that number times 450.

The 10s and 20s columns add up to the 30s column. This is true because of the distributive property. Any number times (20 + 10) is the same as that number times 30. For example, 4(20 + 10) = 4 × 30.

Student page 89

ERROR

Note students who are trying to find each product in the table without using patterns and relationships. To help you determine if they will need extra support, ask them how multiplying by 20 is like multiplying by 2, and how it is different.

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Tell students to continuing working with their partners on problem 3.

As students work, observe their written explanations.

- Are there enough details?
- Are they using math vocabulary?
- Are they precise in their comparisons?

3. How does the multiplication table on page 14 compare to the familiar multiplication table shown here?

Possible answer:

The two multiplication tables are the same except that the factors across the top are 10 times as large, and that makes all the products 10 times as large as their corresponding products in the familiar table.

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

Student page 90

Identify student explanations that you would like to review with the group during the probing for understanding discussion.

probing for understanding

Facilitate discussion of problems 1–3. As students share their explanations, ask the group to confirm, challenge, and justify, or to ask for clarification as needed.

Ask questions to consolidate and extend students' thinking.

- True or false? When I am multiplying 3×20, I can think about 3 groups of 2 tens.

  Answer: True. 20 can be represented as 2 tens.

- How could we write this calculation as an equation?

  Answer: $3 \times 20 = 3 \times 2 \times 10$

- What property of operations does this equation show?

  Answer: The associative property.

- How could you find 8×70 without using the table?

  Answers will vary: Strong strategies will use the associative property or distributive property. Example:

  $8 \times 70 = 8 \times 7 \times 10 = 56 \times 10 = 560$

  $8 \times 70 = (5 \times 70) + (3 \times 70) = 350 + 210 = 560$

  $8 \times 70 = (10 \times 70) - (2 \times 70) = 700 - 140 = 560$

scaffolding for success

Make sure you also write the expressions that you say to help students follow what you are saying.

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Make sure the discussion elicits the following points:

- A multiple of 10 can be represented as a number of tens; for example, 60 is 6 tens.
- You can multiply a multiple of 10 by multiplying the number of 10s and then multiplying the result by 10.
- In the base-ten system, the value of each place is 10 times the value of the place to the immediate right. So, multiplying by 10 results in a product in which each digit of the number being multiplied shifts one place to the left. For example, $10 \times 60 = 600$.
- The associative property can be used when you are multiplying multiples of 10.
- When you are multiplying multiples of 10, the distributive property helps you use products you know to find products you do not know.
- You can use the commutative, associative, and distributive properties to find patterns in the multiplication facts table and in the multiples of 10 table.

Allow students time to edit their answers if they need to add details or want to add an additional sentence after hearing the discussion.

Note that in identifying the patterns and relationships, students use the properties of operations that will also help them understand multiplying by powers of 10 and multidigit multiplication.

reflection

When you have about 2 minutes left, have students respond to the reflection prompt.

reflection

When I multiply by a multiple of ten, I usually …

Student page 90

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How Much Is 15 Tens? 18

Mathematical Goals

- Solve problems involving multiples of 10 by modeling with base-10 blocks and using number sense

Misconceptions Addressed

ERROR

- Is unable to transfer between different representations of the same operation
- Does not understand the relative sizes of digits in place-value notation

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD18-1 through MD18-5
✓ Base-10 blocks (1 hundreds block and about 20 tens blocks per student pair)

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- multiplication *(multiplicación)*
- times

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards **MD18-1** through **MD18-5**. During today's show me, students show their understanding of multiplication by finding how many when given a number of groups and the number in each group (both less than 10) and solving basic multiplication problems. Have students answer the following questions on their response boards.

- Show me how many there are in all.
  - **MD18-1** 3 groups of 7
    Answer: 21
  - **MD18-2** 8 groups of 6
    Answer: 48
  - **MD18-3** 7 groups of 7
    Answer: 49
- Show me the answer to the multiplication problem.
  - **MD18-4** 9×8
    Answer: 72
  - **MD18-5** 6×5
    Answer: 30

To finish, have students complete the show me problem on their student pages.

How Much Is 15 Tens? 18

show me

How many are in 4 groups of 6? 24

Student page 91

setting the direction

Have students look at page 46.

Discuss the chart with students.

- What is in each of the three columns?
  Answer: The first column states a number of tens. The second column shows that number of tens using the hundreds "flat" base-10 block. The third column shows the value of that number of tens.

setting the direction

Fill in the last column of this chart.

10 tens		100
20 tens		200
30 tens		300
40 tens		400
50 tens		500
60 tens		600
70 tens		700
80 tens		800
90 tens		900

Student page 91

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- What does the first row show?

  Answer: 10 tens make 100, so it shows one of the hundreds blocks and a total of 100.

- If 10 tens make 100, what do 20 tens equal?

  Answer: 200

- What pattern do you see in the chart?

  Answer: Each row has an additional 10 tens, so it is 100 more than the row before.

Have students fill out the third column of the chart. Quickly check their entries.

Read the questions below the chart to students and discuss them as students record the answers.

How many tens make 300? 30 How many tens make 500? 50

20 × 10 = 200

Student page 91

- How can you use the chart to answer these questions?

  Answer: Look in the third column and then look across the row at the first column. Answering these questions is the reverse of filling in the chart where you know the number of tens and find the total. Now you know the total and find the number of tens.

- How can you find how many tens are in 300 without looking at the chart?

  Answer: You know that there are 10 tens in a hundred, so there are three times that many tens, 30 tens, in 300.

- How much is 20 × 10? How would you explain your answer to someone without using the chart?

  Answer: 20 × 10 is 200. If you know that 10 tens is 100, then you need another 10 tens to make another 100.

english language learners

For ELLs, explaining the meaning of key vocabulary, such as *times* and *times as many*, is critical, since they are learning a new language as well as new mathematics.

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work time

Presenting the Task

Have a student read problem 1 to the group.

Display base-10 blocks: several hundreds blocks and about 20 tens blocks.

- How can you use base-10 blocks to model a box of pencils?

Make sure students recognize that a "long" can represent 10 pencils, the number of pencils in one box.

work time

1. Anna's class had 15 boxes of pencils. Each box had 10 pencils.

Anna asked:

"How much is 15 tens?"

Answer her question. Use base-10 blocks to model 15 tens.

Draw and write below to explain your thinking.

Sample answer:

15 tens is 10 tens and 5 tens.
That is 100 and 50, which is 150.

Student page 92

Make base-10 blocks available and have students work with partners to complete problem 1 using the partner work ritual.

ERROR

Watch for students who are "adding a zero" as a multiplication trick, without understanding. For example, in problem 1 they might say that you just take 15 and add a zero. When asked, they may not be able to explain why this trick works because they have not had an opportunity to make it make sense. Use partner work and conferencing time to pose questions and have students model with blocks to check students' understanding that in our number system each place-value position to the left represents "ten times as much."

english language learners

The work time problems are an opportunity for ELLs to develop their literacy skills because they incorporate reading, writing, listening, and speaking skills. Encourage students to challenge each other's ideas and justify their thinking using mathematical language.

probing for understanding

Ask the following questions to get at student thinking and promote discussion:

- Who wants to share their explanation? How much is 15 tens?

  Answer: 15 tens is 10 tens and 5 tens. That is the same as 100 and 50, which is 150.

scaffolding for success

Make sure you provide wait time and acknowledge student responses, both verbally and with gestures.

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work time

Presenting the Task

Have a student read problem 2 to the group.

- Who can state the problem in their own words?

Any statement should identify the information needed to solve the problem: Amir had 3 stacks, each of which had 40 trays. How many trays is that in all?

- How might you model this problem to help solve it?

2. Amir helped in his day camp's kitchen after lunch. He made 3 stacks of trays. Each stack had 40 trays.

Amir wanted to know how many trays he had stacked in all.

Tamika started to explain:
"Each stack has 4 tens. So you stacked 4 tens 3 times."

Try to understand Tamika's thinking. Finish her explanation.

How many trays did Amir stack?

A stack has 4 tens, so 3 stacks has 12 tens. 12 tens is 10 tens and 2 tens. That is 100 and 20, which is 120.

Student page 93

Students may suggest using base-10 blocks, making a math drawing using squares and sticks to represents hundreds and tens, or making a drawing of 3 stacks of 40 trays.

Have students continue to work with their partners to complete problem 2.

As you observe students, make note of any mathematical practices that you observe students using so that you can highlight these mathematical practices during the probing for understanding discussions.

probing for understanding

Facilitate discussion of the problem by inviting students to share their explanations.

- How did you finish Tamika's explanation?

  Answer: A stack has 4 tens, so 3 stacks has 12 tens. 12 tens is 10 tens and 2 tens. That is 100 and 20, which is 120.

- Why do you think we started this lesson with the chart in setting the direction?

  Answer: Knowing that 10 tens makes a hundred helps you think about problems that involve more than 10 tens. You can think about 20 tens, 30 tens, and so on. And you can also think about amounts "in between" multiples of 10, such as 14 tens or 17 tens.

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reflection

Writing the Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

reflection

One new thing I learned today is …

Student page 93

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How Many Tens? 19

Mathematical Goals

- Identify numerical expressions that represent sets of objects grouped in multiples of 10, using number sense and properties of operations

Misconceptions Addressed

ERROR

- Does not recognize or misapplies the associative property
- Is unable to transfer between different representations of the same operation
- Does not understand the relative sizes of digits in place-value notation

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD19-1 through MD19-5
✓ Poster paper (one sheet per student pair)

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- expression *(expresión)*
- equal *(igual)*
- equivalent *(equivalente)*
- equation *(ecuación)*

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards **MD19-1** through **MD19-5**. During today's show me, students show their understanding of the associative property by using it to write equivalent multiplication expressions. Have students answer the following questions on their response boards.

- Show me the missing number.
  - **MD19-1** $(2 \times 2) \times 4 = ___ \times (2 \times 4)$
    Answer: 2
  - **MD19-2** $(2 \times 3) \times 3 = 2 \times (3 \times ___)$
    Answer: 3
  - **MD19-3** $(4 \times 2) \times 2 = 4 \times (___ \times 2)$
    Answer: 2
  - **MD19-4** $(2 \times ___) \times 3 = 2 \times (2 \times 3)$
    Answer: 2
  - **MD19-5** $(___ \times 3) \times 2 = 3 \times (3 \times 2)$
    Answer: 3

scaffolding for success

Check that students understand the use of parentheses, and explain that parentheses tell us which numbers to multiply first. Work a problem with students before asking them to answer on their own.

To finish, have students complete the show me problem.

How Many Tens? 19

show me

Write the missing number. $(2 \times 3) \times 5 = 2 \times (3 \times \underline{\ 5\ })$

Student page 95

setting the direction

Have students look at page 49.

Read the first part of the problem as students follow along.

- How does 3×40 represent the problem?
  Answer: Each box has 40 packs and there are 3 boxes the teacher plans to buy. She want to know how many packs that is.

Invite a student to read the next part of the problem.

setting the direction

At the art store, a box of clay has 40 packs of clay.

Josh's teacher has enough money to buy 3 boxes.

She asks the class how many packs of clay there are in 3 boxes.

Josh says:

"We can think about how many tens there are.

Each box has 4 rows of 10 ... (4 × 10)

... and there are 3 boxes." 3 × (4 × 10)

Then he says:

"There are 12 tens because 3 times 4 is 12."

He writes:

(3 × 4) × 10 = 12 × 10

Do you agree with him?

Student page 95

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Facilitate discussion of Josh's reasoning, encouraging students to consider each other's thinking.

Josh is correct. To help students follow his explanation and reason about it, point to each of the expressions in Josh's writing as you write this equation for students to see: $3 \times (4 \times 10) = (3 \times 4) \times 10$

- How can we use the picture of the boxes of clay to help us make sense of this equation?

Facilitate discussion and elicit this explanation in students' own words:

- You can look at one box as 4 rows of 10 packs, for 40 packs, and then find out what 3 times that much is because there are 3 boxes. You find out how much 3 times 40 is. It is 120.
- Or, you can look at all 3 boxes and say there are 3 boxes with 4 rows each, so there are 12 rows. Then you can multiply the 10 packs in a row by 12 because there are 12 rows. You find out what 12 times 10 is. It is 120.

It does not matter which way you do it. Both ways include all the packs, so you get the right total.

teaching strategies

Although students are using the associative property [$a \times (b \times c) = (a \times b) \times c$], they do not need to know the name. The focus of instruction should be these goals:

- It makes sense to students that changing the order in which they multiply numbers does not change the result.
- Students can apply this understanding in thinking about computations.

work time

Presenting the Task

Read the directions for problems 1 and 2 as students follow along.

Make sure students understand the example.

- Why do these five cards go together?

  Answer: The first card is a picture, from the problem students looked at in setting the direction, of groups that are made of groups of 10: There are 3 groups (3 boxes) and each group (box) is made of 4 groups of 10 (4 rows of 10 packets). The other cards show different, and equivalent, ways to represent the picture numerically.

work time

1. Work with a partner.
   - Get pages 97 and 99 and cut out all of the cards.
   - Put the cards in groups. Each group should show the same problem in five ways.
   - When you have made all of the groups, put each group in a row like this one.

3 × 40	3 × (4 × 10)	(3 × 4) × 10	12 × 10

   - Compare your answers with your partner's answers. If they are not the same, try to figure out why.
   - Have your teacher check your groups. Your teacher will tell you when you can tape the cards together.
2. If you have time, make your own row of cards like the ones from problem 1.

Student page 96

english language learners

As you are explaining the directions, rephrase them to provide multiple opportunities for understanding.

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Read problem 2 and explain that when partners have finished problem 1, students should work on problem 2 doing solo work.

Now have students work with partners.

Have each pair carefully cut out all the cards on student pages 97 and 99.

Ask partners to work together to find groups of five cards that go together, just as was done in the example. As they do so, they should explain to each other how they know that the cards match. They should either challenge the explanations or explain why they agree.

When partners are happy with their final groups, have them check with another pair of students to verify their work. Then, ask them to invite you to check the groups. If the cards are correctly grouped, have the partners attach the cards to poster paper.

Student pages 97 and 99

scaffolding for success

Be sure to offer students a variety of ways to access the mathematics, both visually and contextually. Special needs students benefit greatly from tasks, such as card sorts, in which they match different representations of the same mathematical information. In pairs or groups, students can also create posters that show their understanding of mathematical concepts and require visual organization of concepts.

probing for understanding

Ask the following questions to get at students' thinking:

- How did you decide which cards went together?

Take all responses. A student might, for example, suggest taking a picture card and finding the card with numbers on it that most obviously goes with the picture, and then going from there to find the other cards.

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- Who wants to show us a row of cards you made?

Have students display rows of cards, and ask the rest of the group to explain why all the cards go together, or ask a question if they think a card does not belong in the set (is incorrect).

Card Sort Solution

	3×50	$3 \times (5 \times 10)$
	$(3 \times 5) \times 10$	15×10
	4×20	$4 \times (2 \times 10)$
	$(4 \times 2) \times 10$	8×10
	5×30	$5 \times (3 \times 10)$
	$(5 \times 3) \times 10$	15×10

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reflection

Writing the Reflection

When you have about 2 minutes left, have students respond to the reflection prompt.

reflection

Thinking "How many tens?" to answer problems like 6 × 30 helps me because …

Student page 96

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Using Equations to Multiply Tens

20

Mathematical Goals

- Multiply a multiple of 10, decomposing the multiplier and applying properties of operations and number sense

Misconceptions Addressed

ERROR

- Is unable to transfer between different representations of the same operation
- Does not recognize or misapplies the distributive property
- Does not understand the relative sizes of digits in place-value notation

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD20-1 through MD20-5

Optional Materials

✓ Base-10 blocks
✓ Pink eraser for demonstration

Focus on Language

Model the use of this word and encourage students to use it throughout the lesson. Consider displaying the word so students can see it as they work. Spanish cognates are in parentheses.

- product *(producto)*

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards **MD20-1** through **MD20-5**. During today's show me, students demonstrate their understanding of multiplication by finding the product of two single-digit numbers. Have students answer the following questions on their response boards.

- Show me the product.
  - **MD20-1** 3×9
    Answer: 27
  - **MD20-2** 6×7
    Answer: 42
  - **MD20-3** 7×8
    Answer: 56
  - **MD20-4** 8×8
    Answer: 64
  - **MD20-5** 8×9
    Answer: 72

english language learners

Note that the word *product* may be new to students. You may want to briefly discuss the meaning.

To finish, have students complete the show me problem.

Using Equations to Multiply Tens 20

show me

Write the products. $5 \times 7 =$ 35 $1 \times 7 =$ 7 $6 \times 7 =$ 42

Student page 101

setting the direction

Have students look at page 101.

Read the first sentence and direct students to study the picture. Explain that each small square represents one eraser.

- How are the erasers packed in a box?
  Answer: There are 5 rows of erasers, and each row has 10 erasers.
- How does this picture represent the problem?
  Answer: Each rectangle represents a box of 50 erasers. There are 7 rectangles for the 7 boxes of erasers.

setting the direction

The supply room at Anna's school had 7 boxes of 50 erasers.

Anna wanted to find 7×50.

She thought:

"How many tens is that?"

There are 5 tens in each box. (5×10)

There are 7 boxes. $7 \times (5 \times 10)$

That equals 35 tens because 7×5 is 35. $(7 \times 5) \times 10$ 35×10

Do you agree with Anna?

Student page 101

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Guide the group through Anna's solution for finding the total number of erasers, facilitating discussion to allow students to make sense of each step in Anna's thinking.

scaffolding for success

Be sure to point to parts of the picture and the numerical expressions as you discuss them.

Write for students to see: $7 \times (5 \times 10)$ $(7 \times 5) \times 10$

- Anna has both of these expressions in her solutions. Do you agree that both expressions represent the problem? Who wants to use the picture to explain how you know that these expressions are equal?

Facilitate discussion and elicit this explanation in students' own words:

- You can look at one box as 5 rows of 10 erasers, for 50 erasers, and then find out what 7 times that much is because there are 7 boxes. You find out how much 7 times 50 is. It is 350.
- Or, you can look at all 7 boxes and say there are 7 boxes with 5 rows each, so there are 35 rows. Then you can multiply the 10 erasers in a row by 35 because there are 35 rows. You find out what 35 times 10 is. It is 350.

It does not matter which way you do the problem. Both ways include all of the erasers, so you get the same total.

Confirm that students agree with Anna.

work time

Presenting the Task

Read problem 1 as students follow along.

work time

1. Anna thought:
"I can find 35 × 10 the way I have done other problems like this. I'll show my thinking with equations."
Look at how she started. Finish her work.

$35 \times 10 = (30 + 5) \times 10$
$= (30 \times 10) + (5 \times 10)$
$= 300 + 50$
$= 350$

Student page 102

Walk through this recording of the equations with the group. Elicit the following ideas:

- The equation at the top shows how Anna starts to think about the problem. She knows that she can think about 35 as 30 + 5, and that will help her multiply because it is easy to find 30 times 10 and 5 times 10.
- On each line, the expression to the right of the equals sign shows a new way of working with the expression on the line above.
  - What does this expression tell us about the next step in Anna's thinking about the expression in the first line?
    This expression show that Anna is going to think about 30 times 10 and then 5 times 10.

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- All the expressions to the right of the equals signs are equivalent.
  - What do you want to write next? What is your next step, thinking about the expression above, $(30 \times 10) + (5 \times 10)$?
    Have students suggest writing 300 (for 30×10) plus 50 (for 5×10). Have them write $300 + 50$ on their student pages.
  - What is the last thing for you to write?
    The last thing to write is the sum, 350.

Take the time to look with the group through all the expressions to the right of the equals signs, starting at the top, and have students confirm that all the expressions are equivalent.

- All the expressions on the right are equal to the expression on the left of the equals sign in the top line.
  - Why don't we need to rewrite that expression on every line?

    Answer: This expression doesn't change. As we work on the problem and write the expressions on the right, we understand that we are still thinking about that expression on the left.

Read problem 2 as students follow along.

Tell students that they will work solo on the problem for about 1 minute and then work on it with a partner.

2. Josh's class set up chairs for a school play. The students set up 8 rows of 40 chairs. How many chairs did they set up?

Think: How many tens are there in 40? 4 tens

How many tens in 8 rows of 40? 32 tens

Finish the equations to find the number of chairs the class set up.

32 × 10 = (30 + 2) × 10

= (30 × 10) + (2 × 10)

= 300 + 20

= 320

Student page 102

teaching strategies

Having students work in pairs allows you to learn about students' thinking through listening and taking notes on student conversations.

scaffolding for success

Some students may benefit from drawing a picture or using base-10 blocks (each tens block represents 10 chairs, and 4 blocks make a row) to represent this problem. A strong representation would show the 8 rows with 4 groups of 10 chairs:

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probing for understanding

Elicit students' thinking as you facilitate discussion of problem 2 and record each line of the solution for students to see and refer to together.

- How did you complete the first equation? How did splitting 32 into 30 and 2 help you solve the equation?

  Answer: I know how to think about 30 times 10 and 2 times 10, and I can do that in my head without making a mistake.

Continue to record as students explain how they finished the equations.

- Is this way of recording your thinking helpful?

reflection

Writing the Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

reflection

One new thing I know about writing equations is …

Student page 102

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Times Tens Concentration 21

Mathematical Goals

- Identify representations related to multiplying one-digit whole numbers by multiples of 10 and multiples of 100

Misconceptions Addressed

- Confuses place value in whole numbers
- Does not understand the relative sizes of digits in place-value notation

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD21-1 through MD21-5

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards **MD21-1** through **MD21-5**. During today's show me, students show their understanding of place value by stating the number of tens in a number that is a multiple of 10. Have students answer the following questions on their response boards.

- Show me how many tens are in each number.
  - **MD21-1** 20
    Answer: 2
  - **MD21-2** 90
    Answer: 9
  - **MD21-3** 60
    Answer: 6
  - **MD21-4** 80
    Answer: 8
  - **MD21-5** 50
    Answer: 5

To finish, have students complete the show me problem.

Times Tens Concentration 21

show me

How many tens are in 70? 7

Student page 103

setting the direction

Have students look at page 103.

Read through the Simon Says game directions and example as students follow along.

Ask the following questions to make sure students understand the rules:

- When do you respond to what I say?
  Answer: When you say "Simon says …"
- When do you not respond?
  Answer: When you don't say "Simon says …"

setting the direction

Play **Simon Says How Many Tens?**

How to play:

- The teacher speaks.
- If the teacher says "Simon says," you say the correct answer.
- If the teacher does not say "Simon says," you do not answer. If you do, you are out for the rest of the game.
- The winner is the last player still in the game or all players who are still in the game when time is called.

example

Teacher:	"Simon says 2 × 40."
Students:	"8 tens"
Teacher:	"Simon says 150."
Students:	"15 tens"
Teacher:	"4 × 50"
Students:	(Shhh!)

Student page 103

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Play the game until only one player is left in the game or you run out of time for setting the direction.

- Did you get better at responding as the game went along?

The point of this question is to prompt students to think about how practice and repetition may help them develop skills.

work time

Presenting the Task

Tell students they will be playing a game called *Times Tens Concentration*.

Invite a student to help you demonstrate how the game is played.

Read the directions as you demonstrate.

- Spread out a set of the Times Tens Concentration cards face down in a three-by-six array.
- Choose two cards and ask the group to tell you whether the cards match.
- Keep the cards if they match. Return them to the array face down if they don't match.

Now have the student take a turn.

Continue to demonstrate the game until at least one match has been made and you are sure all students understand the directions.

work time

1. Play **Times Tens Concentration** with one or two other players.
   - Get pages 107–112 and cut out all of the cards.
   - Place the cards face down and mix them up. Then make them into a stack.
   - Use the top cards from the stack to lay out a three-by-six array.

   - Players take turns. On each turn, a player flips over any two cards.
     - If the cards match, the player keeps both cards and puts two new cards in the empty spaces of the array. It is now the next player's turn.

     - Other players may challenge the match, explaining why they disagree. The first player must defend the match or agree that the cards do not match.
     - If the cards do not match, the first player turns both cards back over.
   - The winner is the person who has the most cards when all of the cards have been used or time is called.

Student page 104

teaching strategies

Note that not all of the matches are the same kind. The representations of the number are presented in four different forms: picture, multiplication problem, number of tens, product. One match might be a picture and a number of tens; another match might be a product and a number of tens. This activity helps students develop facility with different representations.

english language learners

Modeling the activity step by step will ensure that ELLs understand the task and can focus on the mathematics of the activity rather than on trying to figure out the directions.

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Have students look at the example on their student pages.

- Why are these two cards a match?

  Answer: These cards match because 2 × 30 is the same as 2 times 3 tens, which is 6 tens.

Have students play the game with one or two other students. Each group will need to cut out one set of Times Tens Concentration cards (student pages 107–112).

probing for understanding

Facilitate a discussion of students' experiences playing the game.

- Did you ever disagree about a match? How did you decide whether the cards matched?

Have students give examples.

- Were some forms of the number (picture, multiplication problem, number of tens, product) easier to work with?

Keep in mind that there are no right or wrong answers here. Some students may find one form easy to work with, and others may find the same form more difficult to work with.

scaffolding for success

It is critical that students feel safe enough to share their ideas, strategies, and, most important, mistakes. Most students—and special needs students in particular—are not used to talking about mathematics; yet in higher-level courses, student success might depend on participation in study groups and the student's ability to share ideas.

work time

teaching strategies

You may want to have students who finish the concentration game early start on this problem or you may want to have the group work together on this problem. Multiplying by multiples of 100 goes beyond the goals of this module. It is included here to prompt students to notice important features of our base-ten number system: The value of a digit depends on the place it is in; a digit in one place represents ten times what it represents in the place to its right; only one digit can be written in each place; sometimes we need a placeholder, and we use zero as that placeholder.

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Presenting the Task

Have students read problem 2 quietly to themselves.

Students should work solo on the problem for about 1 minute and then work with a partner.

2. You know how to multiply ones. You know how to multiply tens.

What do you know about multiplying hundreds?

Look at the table below. Fill in the blanks.

Hundreds	Tens	Ones
4 × 200	4 × 20	4 × 2
8 hundreds	8 tens	8 ones
800	80	8

Student page 105

probing for understanding

Facilitate discussion of problem 2.

Elicit these observations and ideas from students:

- The product is always 8 of something. It is 8 ones, or 8 tens, or 8 hundreds.
- The product is always 8 because you start with 2 of something—ones, tens, or hundreds—and multiply by 4, and 4 times 2 of something is always 8 of something.
- Each place represents 10 times what the place to its right represents. A ten is 10 times a one. A hundred is 10 times a ten.
- So, 20 is 10 times 2: For each of the ones blocks, we have a tens block in the Tens column.
- And 200 is 10 times 20: For each of the tens blocks, we have a hundreds block in the Hundreds column.
- And 800 is 10 times as many as 80. And 80 is 10 times as many as 8.
- There is a pattern in the zeroes in the products: 8, 80, 800. That is because the value of the 8 is ten times more as you go from 8 ones, to 8 tens, and then to 8 hundreds.
- The zeroes tell us what place the 8 is in. That is why we call zero the placeholder in our number system. Without a placeholder, we would not know whether the digit 8 means 8 or 80 or 800. Using the zeroes, we can show what place 8 is in.

reflection

Writing the Reflection

When you have about 2 minutes left, have students respond to the reflection prompt.

reflection

Multiplying tens is (circle one) ☺ ☹ for me because …

Student page 106

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Solving Problems with Multiples of 10 and 100

22

Mathematical Goals

- Multiplies multidigit whole numbers using strategies based on place value and properties of operations, in particular the distributive property

Misconceptions Addressed

ERROR

- Treats each place value of a whole number separately when multiplying and dividing
- Does not recognize or misapplies the distributive property

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD22-1 through MD22-5

Optional Materials

✓ Chart paper
✓ Sample book of raffle tickets for demonstration purposes (real or simulated by stapling small slips of paper together)

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD22-1 through MD22-5. During today's show me, students will write an equation that uses the distributive property and related facts to find a basic multiplication fact. Have students answer the following questions on their response boards.

- Show me how to fill in the blanks to make this equation true.
  - $3 \times 7 = (3 \times ____) + (3 \times ____)$

    Answer: Answers will vary. Accept any two numbers that sum to 7; for example, $(3 \times 5) + (3 \times 2)$.
  - $5 \times 11 = (5 \times ____) + (5 \times ____)$

    Answer: Answers will vary. Accept any two numbers that sum to 11; for example, $(5 \times 1) + (5 \times 10)$.
  - $8 \times 6 = (8 \times ____) + (8 \times ____)$

    Answer: Answers will vary. Accept any two numbers that sum to 6; for example, $(8 \times 5) + (8 \times 1)$.
  - $7 \times 9 = (7 \times ____) - (7 \times ____)$

    Answer: Answers will vary. Accept any two numbers that have a difference of 9; for example, $(7 \times 10) - (7 \times 1)$.
  - $9 \times 12 = (9 \times ____) + (9 \times ____)$

    Answer: Answers will vary. Accept any two numbers that sum to 12; for example, $(9 \times 10) + (9 \times 2)$.

To finish, have students complete the show me problem.

Solving Problems with Multiples of 10 and 100 22

show me

Show me how to fill in the blanks to make this equation true.

$5 \times 8 = (5 \times ____) + (5 \times ____)$ Accept any two numbers that sum to 8.

Student page 113

scaffolding for success

This approach to the multiplication facts not only develops number and operation sense but also gives students a strategy for finding any fact they cannot recall.

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setting the direction

Have one student read the setting the direction problem aloud. Ask:

- Who would like to restate this problem in his own words?

Take one or two responses. Ask students if they agree with the restatement, need clarification from the speaker, or have a suggestion for more clearly communicating what the problem is about and what it is asking.

- Who wants to tell us how you would solve this problem?

As a student suggests an approach, record it on the board or chart paper so students can follow the explanation. As you record, model asking the speaker questions for clarification, if needed. For example, you might ask, "How did you know $14 \times 3 = 42$?"

setting the direction

Anna read 30 pages of her favorite book last night. She wonders how many pages she will read if she reads 30 pages each night for two weeks. (There are seven days in a week.) Find one way to solve this problem.

Write the answer in a complete sentence.

Anna will read 420 pages in 14 days.

Student strategies will vary and may include equations, diagrams, and tables.

Possible answers:

$14 \times 30 = 14 \times 3 \times 10$
$= (14 \times 3) \times 10$
$= 42 \times 10$
$= 420$

$14 \times 30 = 14 \times 3 \times 10$
$= (14 \times 10) \times 3$
$= 140 \times 3$
$= (100 + 40) \times 3$
$= 300 + 120$
$= 420$

$30 \times 14 = 30(10 + 4)$
$= 300 + 120$
$= 420$

Student page 113

- Who agrees with this solution? Does anyone have a question for [name of student who presented the strategy]?
- Who did something different?

teaching strategies

As students become fluent in using strategies that are based on place value and properties of operations, especially the distributive property, they are laying the foundation for learning the standard multiplication algorithm. The partial products method that students use here can be connected later with the standard algorithm.

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work time

Ask students to look at page 114. Read problem 1 as students follow along.

Tell students they will work on all parts of problem 1 doing partner work. Partners may have different strategies for solving the problem. They should write their own strategies on their student pages.

As students work, note partners who use different computation strategies so you can include them during probing for understanding.

work time

1. Tamika's school sets up 300 chairs for each play performance. There will be 13 performances this year. Tamika helps set up chairs. She wonders how many chairs will be set up in all.

Find one way to solve this problem.

3,900 chairs will be set up.

Write the answer in a complete sentence.

Student strategies will vary and may include equations, diagrams, and tables.

Possible answers:

$13 \times 300 = (10 + 3) \times 300$
$= (10 \times 300) + (3 \times 300)$
$= 3{,}000 + 900$
$= 3{,}900$

$300 \times 13 = 300(10 + 3)$
$= (300 \times 10) + (300 \times 3)$
$= 3{,}000 + 900$
$= 3{,}900$

Student page 114

english language learners

Remember that word problems tend to be difficult for ELL students because they are highly language dependent. In addition, many word problems require formal operations or the ability to think abstractly and to manipulate concepts through language. Encourage students to work with a partner and to share and exchange ideas as they are working through difficult word problems.

ERROR

Note any students who have difficulty recognizing or applying the distributive property in today's lesson. They may need additional experiences decomposing and composing numbers in order to become confident that numbers can be broken apart.

probing for understanding

- Did your partner solve the problem the same way you did or a different way?

Have two or three students share different strategies. Record their strategies so other students can follow along and so the strategies are available for reference.

- Do both ways work? If so, explain why the different ways both solve the problem.

  Answers will vary: Variations will usually have to do with how students have broken apart (decomposed) the numbers.

If two students have used the same strategy, with differences only in the way they have written the computations, help students clarify that the strategies are actually the same.

- Did either of these strategies use the distributive property? Where do you see that?

  Answers will vary: Have students point out all the places where the distributive property was used.

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work time

Have a student read both parts of problem 2 aloud to the group.

english language learners

Some students may not be familiar with raffles. Have a sample of a book of raffle tickets available so that students can understand the context of the problem. Point to the appropriate parts of the book of tickets when discussing the problem.

2. a. Anthony's school group is printing raffle tickets. The group is planning to put 20 tickets in a book. How many tickets will they have if they print 40 books?

Find one way to solve this problem.

They will have 800 tickets.

Student strategies will vary and may include equations, diagrams, and tables.

Possible answers:

$20 \times 40 = 10 \times 2 \times 40$
$= (10 \times 40) \times 2$
$= 400 \times 2$
$= 800$

$20 \times 40 = 20 \times 4 \times 10$
$= (20 \times 4) \times 10$
$= 80 \times 10$
$= 800$

b. The group wants to print 1,200 raffle tickets. Now Amir knows that 40 books will not be enough. How many books (of 20 tickets) does the group need to print?

Find one way to solve this problem.

The group will need to print 60 books.

Student strategies will vary and may include equations, diagrams, and tables. Possible answer:

40 books make 800 raffle tickets. 1,200 tickets is 400 more. That's half of 800. So half of 40 books is the rest of what they need. That's 20 books. 40 books + 20 books = 60 books

Student pages 114–115

Tell students to continue to work on problem 2 with their partners.

Observe students as they work. Note at least two different strategies being used for each problem, so you can ask students to share them during probing for understanding. If you notice misconceptions as you observe students' written work and hear their conversations, plan to address these in the discussion as well. Students must solve this problem by representing quantities in the situation with equations, diagrams, or tables. To do this, students must mathematize a real-world situation, focusing on the quantities and their relationships rather than on the non-mathematical aspects of the situation.

probing for understanding

Discuss students' strategies for solving part a. Note that, for the first time in this module, students are asked to multiply with two multiples of 10. Take the time to highlight how students decomposed the factors 20 and 40 in the strategies they share.

Have a student record his computation strategy on the board or chart paper as he explains his solution.

Have a student who used a different strategy present her work in the same way.

Encourage students to compare and contrast the strategies presented, agree with or challenge the solutions shared, and justify their responses.

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Now lead a discussion on problem 2, part b.

- How did you think about part b in order to understand what the problem was asking?

  Answers will vary: Note that this problem and problem 3 require that students understand the need to find a missing factor. Ask questions, if needed, to help students explain the reasoning they used to understand this problem.

- How did you approach solving the problem?

  Answers will vary: Some students may have asked, "What number times 20 will make 1,200?" Other students may have used what they know from part a, along these lines: 400 is half of 800. I know that 40 books makes 800 tickets, so I need half that many books again, or 20 more books.

As students describe their strategies for solving the problem, record on the board or chart paper for students to follow along. Allow time for students to ask the presenters questions about their solutions.

work time

Read problem 3 aloud to students.

Tell students that they will work on problem 3 doing partner work.

As partners discuss their ideas for solving the problem, monitor their thinking and make note of any points you feel you will need to focus on during probing for understanding.

3. If the raffle tickets are printed in books of 30 tickets instead, how many books will the group need in order to have 1,200 tickets?

The group will need 40 books.

Student strategies will vary and may include equations, diagrams, and tables. Possible answer:

I know that 3 × 4 = 12, so 30 tickets × 4 books = 120 tickets.

The group needs 10 times that much—1,200. So, the group needs 40 books. 30 × 40 = 1,200

Student page 115

probing for understanding

scaffolding for success

Extend student responses by using prompts and targeted questions that are appropriate to the students' abilities.

Lead a discussion of problem 3. Note whether students connect this problem with part b of problem 2; in both, students must find a missing factor.

Ask questions to review what students have learned about multiplying multidigit numbers by a single-digit number:

- Do you think you can always use the distributive property when you are multiplying a multidigit number by a single-digit number? Why or why not?

  Answer: It is always possible to use the distributive property because either factor can be broken into other numbers.

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- Let us hear a strategy for finding the product 6×349 without breaking 349 into hundreds, tens, and ones. What is a fast way to break 349 into *other numbers* and solve this problem?

  Answers will vary: One strong approach is: $6(300 + 50) - 6(1) = (6 \times 300) + (6 \times 50) - (6 \times 1)$
  $= 1{,}800 + 300 - 6$
  $= 2{,}100 - 6$
  $= 2{,}094$

teaching strategies

Students should continue to develop their mental computation skills. One way to encourage students' development is to present problems, like the last one here, that can be solved using special strategies that do not generalize to all problems of a certain type and to have students share more than one strategy for solving the problems.

reflection

Writing the Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

reflection

Breaking up numbers helps me solve multiplication problems because …

Student page 115

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Counting and Calculating the Area of Rectangles

23

Mathematical Goals

- Finds the area of a rectangle by multiplying the dimensions—length and width—together
- Understands that many different rectangles have the same area
- Understands that area is expressed in square units

Misconceptions Addressed

ERROR

- Does not know how to find area
- Confuses area and perimeter

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD23-1 through MD23-5
✓ Chart paper

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- column *(columna)*
- row

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD23-1 through MD23-5. During today's show me, students will find all factors of a number. Have students answer the following questions on their response boards.

- Show me all the factors of …
  - … 10.

    Answer: 1, 2, 5, 10
  - … 12.

    Answer: 1, 2, 3, 4, 6, 12
  - … 15.

    Answer: 1, 3, 5, 15
  - … 9.

    Answer: 1, 3, 9
  - … 16.

    Answer: 1, 2, 4, 8, 16

To finish, have students complete the show me problem.

Counting and Calculating the Area of Rectangles 23

show me

Show me all of the factors of 20.

1, 2, 4, 5, 10, 20

Student page 117

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setting the direction

- Today we will connect the idea of counting squares to two other ways of finding the area of rectangles.

Read the setting the direction problem aloud.

Tell students that one way to calculate the area of a rectangle is to view the rectangle as being broken into a certain number of "rows of squares," in which each row has the same number of squares. To find the total number of squares, add the number of squares in each row as many times as there are rows. For example, in the first rectangle, there are 5 squares in each of 2 rows, so add 5 + 5.

setting the direction

Instead of counting the squares to find the area of a rectangle, you can calculate the area in two other ways:

Examples:

5 + 5 = 10 square feet
2 × 5 = 10 square feet

5 + 5 + 5 = 15 square feet
3 × 5 = 15 square feet

5 + 5 + 5 + 5 = 20 square feet
4 × 5 = 20 square feet

Calculate the area of the following rectangles using both methods described above.

Student page 117

Then explain that another way to calculate the area of a rectangle is to multiply the number of rows by the number of columns.

Instruct students to work with their partners to complete the problems on the student pages.

After students have worked the problems, ask them to share the ways they calculated each of the areas. Record their solutions on the board or chart paper for all students to see. Make sure everyone agrees with the calculations.

scaffolding for success

Model the use of mathematical language during group discussions by using math terms and rephrasing what students say to use more math language.

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work time

Tell students that today's lesson will focus on developing ways to calculate the area of a rectangle without having to break a diagram of the rectangle into square units.

Read problems 1–3 aloud to students.

Have students work with their partners on all of the problems. They should keep moving through the problems as they complete them.

As students work, spend some time conferencing with one or more students individually.

english language learners

During work time, ELLs have the opportunity to develop their literacy skills, since the problems incorporate reading, writing, listening, and speaking skills. Encourage students to challenge each other's ideas and to justify their thinking using academic and specialized mathematical language.

work time

1. a. Circle the rectangles that have an area of 12 square units.

b. Sketch two other rectangles that have an area of 12 square units and label the side lengths.

2. Using only whole numbers, what are all possible dimensions of a rectangle with an area of 30 square inches?

1 in × 30 in 30 in × 1 in 2 in × 15 in 15 in × 2 in

3 in × 10 in 10 in × 3 in 5 in × 6 in 6 in × 5 in

3. Fill in each blank with the missing dimension.

a.

Area is 21 square inches.

b.

Area is 24 square inches.

c. Explain how you found the missing dimensions.

I figured out what number times the given dimension would give me the area.

Student pages 118–119

ERROR

Watch for students who cannot find all the rectangles with a given area. If students did not find all eight possible rectangles in problem 2, prompt them to think about all of the factors of 30 and to think of all the ways you can multiply two factors together to get a product of 30. Remind them that they can consider a 5 inch by 6 inch rectangle as different from a 6 inch by 5 inch rectangle because the two rectangles would look a little different from one another.

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probing for understanding

Ask questions.

- Which of the rectangles in problem 1a has an area of 12 square units?

  Answer: The 3 by 4 rectangle, the 6 by 2 rectangle, the 1 by 12 rectangle, and the 4 by 3 rectangle each have an area of 12 square units.

- Describe two other rectangles you found for problem 1b that have an area of 12 square units.

  Possible answers: a 2 by 6 rectangle and a 12 by 1 rectangle

When students discuss how they determined their answers to problem 1, make sure they understand that the dimensions of the rectangles are all pairs of factors of 12.

- What was the most challenging part of problem 2?
- I noticed that some students used diagrams to help them understand the task. Can you show us what you did and tell us how it helped you?
- Did anyone try a method that did not work?
- What rectangles did you find for problem 2?

  Answers: 1 inch by 30 inches, 30 inches by 1 inch, 2 inches by 15 inches, 15 inches by 2 inches, 3 inches by 10 inches, 10 inches by 3 inches, 5 inches by 6 inches, and 6 inches by 5 inches

- What answers did you get for problem 3?

  Answer: The first rectangle's missing dimension is 7 inches, and the second rectangle's missing dimension is 6 inches. (Make sure students include the label "inches" when noting the dimensions.)

- How did you find the missing dimension?

  Answer: Students should respond that they took the given area and divided by the dimension they knew in order to find the missing dimension.

reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished. Have students respond to the reflection prompt.

reflection

One thing I know about finding the area of a rectangle is …

Student page 120

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Matching Areas of Rectangles

24

Mathematical Goals

- Connects diagrams of rectangles to their calculated areas
- Connects diagrams of rectangles broken into unit squares to diagrams of rectangles with their dimensions labeled

Misconceptions Addressed

- Does not know how to find area
- Confuses area and perimeter

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD24-1 through MD24-5
✓ Scissors (one per student)
✓ Tape or glue (one per student)

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD24-1 through MD24-5. During today's show me, students will find related multiplication facts for addition expressions. Have students answer the following questions on their response boards.

- Show me the related multiplication problem.
  - $5 + 5 + 5 + 5$

    Answer: 4×5
  - $7 + 7 + 7$

    Answer: 3×7
  - $2 + 2 + 2 + 2 + 2 + 2 + 2$

    Answer: 7×2
  - $4 + 4 + 4 + 4 + 4 + 4$

    Answer: 6×4
  - $10 + 10 + 10 + 10 + 10$

    Answer: 5×10

To finish, have students complete the show me problem.

Matching Areas of Rectangles 24

show me

Show me how to write the sum 9 + 9 + 9 using multiplication.

9 × 3 or 3 × 9

Student page 121

english language learners

When working with ELLs, it is important to link new concepts to students' prior knowledge and experiences. This allows students to make more meaningful connections with the new mathematical concepts being learned.

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setting the direction

Tell students that today's setting the direction focuses on helping students clarify the vocabulary and expressions used when examining dimensions of rectangles and the area of rectangles.

Read the setting the direction problems aloud.

Instruct students to work solo to complete the problems on their student pages.

When students have filled in all of the blanks, ask if any students had difficulty determining what to write in any of the blanks. Review the answers.

Be sure students understand that the dimensions of a rectangle can be named in any order and one dimension is the length and the other dimension is the width.

setting the direction

You can say that the rectangle below has dimensions of either 7 inches by 2 inches or 2 inches by 7 inches. The area of the rectangle is 7 × 2 or 2 × 7, or 14 square inches.

One dimension is the *length* of the rectangle and the other is the *width*.

Fill in the blanks with the correct dimension or area.

If the area of a rectangle is 42 square centimeters and the length is 7 centimeters, then the width of the rectangle is 6 centimeters.

Since a square is a special type of rectangle, a rectangle with dimensions of 5 meters by 5 meters has an area of 25 square meters.

You can say that a rectangle with a length of 15 feet and a width of 3 feet has the dimensions 15 feet by 3 feet or 3 feet by 15 feet.

Student page 121

english language learners

Using visuals is a great strategy because students can associate math terms with something concrete.

work time

Read problems 1 and 2 aloud to students.

Have students work with a partner.

Have each pair find their *Rectangles* and *Areas* pages (student pages 125 and 127) and cut out all of the cards.

Ask partners to work together to find a rectangle card and an area card that match each of the rectangles broken into unit squares on student pages 123 and 124. As pairs work, students should explain to each other how they know that the cards match. They should either challenge explanations or explain why they agree.

work time

1. Work with your partner to match rectangles with their areas.
   - Get the *Rectangles* and *Areas* pages (pages 125 and 127) and cut out all of the cards.
   - Find the rectangle card and area card that match each of the rectangles (broken into unit squares) on pages 123–124. (Note: There will be some extra cards after you are done matching.)
   - When you make a match, explain to your partner how you know that the cards match.
   - Your partner should either agree with your explanation or challenge it if your explanation is not clear, correct, and complete.
   - Once you and your partner agree on all of the matches, ask your teacher to check your matches. Then tape the cards into the correct spaces on pages 123–124.

2. Why can you calculate the area of a rectangle by *either* multiplying the length by the width or the width by the length?

   By the commutative property of multiplication, you can multiply in any order and get the same product.

Student page 122

Observe students as they begin working on the problem. Do not be too quick to offer additional help. Allow individual struggle time. Offer assurance, not assistance.

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As students are working, ask questions such as these:

- How are you approaching looking for matches?
- Did you change your mind about any cards? Which ones? How did you identify your error? What did you learn?
- What makes matching the cards difficult? What makes it easy?

When partners are happy with their final matches, have them invite you to check their cards. If the cards are correctly matched, have the partners tape or glue the cards onto their student pages.

Have students answer problem 2 working solo. Then have them check their explanation with their partner.

Spend some time conferencing with one or more students individually.

Match and then tape rectangle and area cards below:

	TAPE RECTANGLE CARD HERE	TAPE AREA CARD HERE
	TAPE RECTANGLE CARD HERE	TAPE AREA CARD HERE
	TAPE RECTANGLE CARD HERE	TAPE AREA CARD HERE
	TAPE RECTANGLE CARD HERE	TAPE AREA CARD HERE
	TAPE RECTANGLE CARD HERE	TAPE AREA CARD HERE
	TAPE RECTANGLE CARD HERE	TAPE AREA CARD HERE

Student pages 123–124

ERROR

Watch for students who do not connect the diagram with labeled dimensions with the diagram of square grids correctly. Students may simply find a similar shape instead of counting the square grids along a dimension.

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RECTANGLES

6 units, 5 units	7 units, 9 units	12 units, 5 units
9 units, 4 units	4 units, 8 units	4 units, 7 units
7 units, 5 units	10 units, 6 units	5 units, 8 units

Student page 125

AREAS

28 square units	60 square units	30 square units
36 square units	35 square units	32 square units
60 square units	63 square units	40 square units

Student page 127

scaffolding for success

Card-sorting activities encourage students to experiment—students can move the cards around if they change their minds. This type of activity also encourages discussion because students have to explain to each other why they think certain cards match.

probing for understanding

Ask students questions.

- What are the dimensions of the rectangle card for the first rectangle?

  Answer: 5 units by 7 units

- What area card matches it?

  Answer: 35 square units

- How did you find the matching area card?

  Answer: Answers will vary. Some students may have counted squares in the grid diagram, while other students may have multiplied the length by the width.

Continue to go through the remaining rectangles.

Ask pairs to share with the group any difficulties they had matching cards. Probe to see what made it challenging and how they ultimately were able to figure out which cards made a match.

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Card Sort Solution

		35 square units
		60 square units
		30 square units
		60 square units
		32 square units
		36 square units

reflection

Writing a Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

reflection

If I know the dimensions of a rectangle, I can find the area of the rectangle by …

Student page 124

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Using Addition and Multiplication to Find the Area of Rectangles

25

Mathematical Goals

- Understands that rectangles can be broken into rows and columns in which each row is the same size and each column is the same size
- Finds area by adding same-sized rows together or by adding same-sized columns together
- Connects adding same-sized rows (or columns) together to multiplication

Misconceptions Addressed

- Does not know how to find area
- Confuses area and perimeter

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD25-1 through MD25-5
✓ Chart paper

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD25-1 through MD25-5. During today's show me, students will practice multiplying two numbers. Have students answer the following questions on their response boards.

- Show me the product.
  - 7×5

    Answer: 35
  - 8×8

    Answer: 64
  - 4×9

    Answer: 36
  - 10×10

    Answer: 100
  - 9×8

    Answer: 72

To finish, have students complete the show me problem.

Using Addition and Multiplication to Find the Area of Rectangles 25

show me

Show me the solution to: 6×8 = 48

Student page 129

english language learners

When working with ELLs, it is important that you provide multiple opportunities for practice, allow extra time for repetition, and provide positive, inviting feedback.

scaffolding for success

If time permits, it might be a good idea to have students derive the formula for the area of a rectangle. Having students discover this formula themselves helps them conceptually understand how the formula works.

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setting the direction

Tell students that the emphasis today is on seeing rectangles broken into rows and columns.

Read the setting the direction aloud to students.

Have students work with their partners to answer the setting the direction problems.

After students have had a chance to complete all the problems, go through the answers and have different students provide answers.

There are two ways to write each addition and multiplication equation, depending on whether you repeatedly add the number in each row (or repeatedly add the number in each column) or whether you multiply the number of rows by the number of columns (or multiply the number of columns by the number of rows). Be sure to show and discuss how both methods are correct.

setting the direction

The following rectangle is broken into unit squares.

How many *rows* does the rectangle have? 6 rows

How many *columns* does the rectangle have? 9 columns

What addition equation can you write to find the total number of unit squares?

9 + 9 + 9 + 9 + 9 + 9 = 54 or

6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 = 54

What multiplication equation can you write to find the total number of unit squares?

6 × 9 = 54 or 9 × 6 = 54

What is the area of the rectangle? 54 square units

Student page 129

english language learners

Using visuals is a great strategy because students can associate math concepts with something concrete. Sketch the diagram from the problem on chart paper and point to the rows and columns as you talk about them.

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work time

Have students read problems 1–3 aloud to the group.

Review the example that is completed in the first row of the table so that students understand what information is needed for each column.

Tell students they will complete the *Connections Table* with their partner.

Have students find the *Connections Table* page (student page 131). Pairs should work together to fill in the missing information in each column of the table. Partners should explain to each other how they know what the missing information is. They should either challenge explanations or state why they agree. Remind them to be good listeners and ask questions when they are not clear about their partners' explanations.

When pairs are done completing the table, they should move on to problems 2 and 3. Have students work independently at first on problems 2 and 3 and then share their answers with their partner.

scaffolding for success

To check for understanding of the task, have students tell you the instructions in their own words.

Observe pairs as they work and listen to students' explanations. Note any obvious misconceptions that emerge so you can bring them up during the whole-group discussion.

You may wish to spend some time conferencing with one or more students individually.

work time

1. Work with a partner to connect the dimensions of a rectangle, the addition calculation you would use to find its area, the multiplication calculation you would use to find its area, and the area.
   - Get the *Connections Table* page (page 131).
   - The first column of the table shows rectangles broken into unit squares. For each rectangle, fill in the missing information in the other columns. Follow the example provided in the first row.

2. How would you explain to a friend how to find the area of a rectangle using addition?

   You add the length to itself and repeat the addition by the number of times given by the width (or vice versa).

3. How would you explain to a friend how to find the area of a rectangle using multiplication?

   You multiply the length times the width (or vice versa).

Student page 130

CONNECTIONS TABLE

Rectangle	Number of Rows	Number of Columns	Addition Calculation	Multiplication Calculation	Area
	8	4	4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 = 32 or 8 + 8 + 8 + 8 = 32	8 × 4 = 32 or 4 × 8 = 32	32 square units
	6	7	7 + 7 + 7 + 7 + 7 + 7 = 42 or 6 + 6 + 6 + 6 + 6 + 6 = 42	6 × 7 = 42 or 7 × 6 = 42	42 square units
	3	9	9 + 9 + 9 = 27 or 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 27	3 × 9 = 27 or 9 × 3 = 27	27 square units
	5	5	5 + 5 + 5 + 5 + 5 = 25	5 × 5 = 25	25 square units

Student page 131

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ERROR

Watch for students who mislabel the area in units instead of square units. Remind students that area measures two-dimensional space, so the answer should be in square units.

probing for understanding

On chart paper, post a blank version of the *Connections Table*. Have different pairs take turns using their own work to fill in one row of the table for all to see. As each row is completed, ask the rest of the group if they agree with everything in that row. Discuss any disagreements and come to an all-group resolution before moving on to the next row.

As the table is being completed, ask questions such as the following:

- What are you trying to find?
- How can we see the addition equation in the rectangle?
- How can we see the multiplication equation in the rectangle?
- Why did you write the answer using square units?

When you have about 4 minutes left, stop the discussion, even if you are not finished and move on to a discussion of problems 2 and 3. Have students share their explanations.

- What are different ways of finding the area of rectangles broken into unit squares?

  Answers: You can add all the unit squares; you can add the number of unit squares in each row together; you can add the number of unit squares in each column together; you can multiply the number of unit squares in a row by the number of unit squares in a column; or you can multiply the number of unit squares in a column by the number of unit squares in a row.

teaching strategies

The commutative property of multiplication states that the order of numbers does not matter when you are multiplying numbers. There is no fixed way to write the dimensions of an array as rows × columns or columns × rows. Students should be able to describe both dimensions of an array in the discussion.

scaffolding for success

If time permits, suggest that students create graphic organizers that incorporate the major categories in the table within the lesson. Students should create organizers that make sense to them and that clearly outline the key ideas.

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reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished. Have students respond to the reflection prompt.

reflection

Finding the area of a rectangle is easy hard (circle one) for me because …

_____

Student page 133

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Solving Area Word Problems 26

Mathematical Goals

- Recognizes problem situations involving area
- Draws diagrams to represent area situations
- Solves word problems involving area

Misconceptions Addressed

- Does not know how to find area
- Confuses area and perimeter

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD26-1 through MD26-5
✓ Chart paper

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD26-1 through MD26-5. During today's show me, students will find the missing factor in an equation. Have students answer the following questions on their response boards.

- Show me the missing factor.
  - ____ × 5 = 45

    Answer: 9
  - 7 × ____ = 56

    Answer: 8
  - ____ × 6 = 60

    Answer: 10
  - 8 × ____ = 72

    Answer: 9
  - 10 × ____ = 100

    Answer: 10

To finish, have students complete the show me problem.

Solving Area Word Problems 26

show me

Show me the missing factor.

8 × 6 = 48

Student page 135

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setting the direction

Today the formula for finding the area of a rectangle will be reinforced. Students will calculate areas and find missing dimensions of rectangles by thinking of missing factors.

Ask a student to read the setting the direction problems aloud.

Have students complete all of the setting the direction problems working solo, and then check their answers with their partner.

When students have filled in all of the blanks, go over the answers with the whole group. Ask different students to tell what answers they had for each problem.

setting the direction

Mr. Hakim said that you can use a formula to find the area of a rectangle.

Area of a Rectangle Formula
Area = length × width

Use the formula to fill in the blanks below.

If a rectangle has a length of 8 units and a width of 7 units, then the area is 56 square units.

If a rectangle has an area of 72 square units and a length of 8 units, then the width is 9 units.

If a rectangle has an area of 70 square units and a width of 10 units, then the length is 7 units.

Student page 135

- How did you find the missing dimensions in the second and third problems?

  Answers: Answers will vary. Students may think of finding a missing factor or think of a division equation. Some students may sketch a diagram using square units and then count the units.

work time

Have a student read all parts of problem 1 aloud to the group. Tell students that they will complete all the problems in this lesson using partner work. Explain that students will read the remaining problems with their partners.

Students are asked to engage in the mathematical practice of making sense of problem situations and to persevere in solving them. To do this, students represent situations using diagrams. Students must mathematize a real-world situation, focusing on the quantities and their relationships rather than on the nonmathematical aspects of the situation.

work time

1. a. In the game of checkers, a checkerboard has 8 rows of squares with 8 squares in each row. Sketch a picture of a checkerboard.

b. Write a multiplication equation that you can use to find the total number of squares on a checkerboard. How many squares are on a checkerboard?

$8 \times 8 = 64$

There are 64 squares on a checkerboard.

c. If each square on a checkerboard is 1 inch by 1 inch, what is the area of the checkerboard?

8(1 inch) × 8(1 inch) = 8 inches × 8 inches = 64 square inches
The area of the checkerboard is 64 square inches.

Student page 136

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Spend time conferencing with one or two students during this time.

ERROR

Watch for students who add the length and the width to find the area instead of multiplying the length and width. These students may need more work finding area with square units to understand the area concept. Have students use a variety of physical materials, such as square color tiles, to fill in a space in order to illustrate area as the space within an enclosed shape. Students should experience the act of finding the space within an enclosed area through actual measurements in order to reinforce the meaning of area without using formulas. This also solidifies the need to label area measures as square measures.

english language learners

Remember that word problems tend to be difficult for ELLs because the problems are highly language dependent. In addition, many word problems require formal operations or the ability to think abstractly and to manipulate concepts through language. Encourage students to work with a partner and to share and exchange ideas as they are working through these problems.

2. a. A rectangular patchwork quilt has 9 rows of square patches with 7 patches in each row. Sketch a picture of what the quilt might look like.

b. Write a multiplication equation you can use to find the total number of squares in the quilt. How many squares are in the quilt?

$7 \times 9 = 63$ or $9 \times 7 = 63$

There are 63 squares in the quilt.

c. If each square in the quilt is 1 foot by 1 foot, what is the area of the quilt?

7(1 foot) × 9(1 foot) = 7 feet × 9 feet = 63 square feet or 9(1 foot) × 7(1 foot) = 9 feet × 7 feet = 63 square feet

The area of the quilt is 63 square feet.

3. a. A rectangular photograph is 8 inches wide and 10 inches long. Sketch and label a picture of the photograph.

b. Write a multiplication equation that you can use to find the area of the photograph. What is the area of the photograph?

8 inches × 10 inches = 80 square inches or

10 inches × 8 inches = 80 square inches

The area of the photograph is 80 square inches.

Student pages 137–138

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english language learners

You may need to model problem 4 to help students understand what they are supposed to do.

4. a. Think of a situation about a rectangle that can be broken into equal-sized squares and describe it below.

Possible answer: Mr. Hakim is covering his rectangular patio with 1-foot square tiles.

b. Share your idea with your partner.

c. With your partner, choose one of the ideas. Then write a word problem that involves finding the area of the rectangle in the situation. (You do not need to solve the problem yet.)

Possible answer: Mr. Hakim is covering his rectangular patio with 1-foot square tiles. His patio measures 10 feet wide by 9 feet wide. What is the area of his patio?

d. Sketch a picture that represents your situation.

Possible answer:

e. Find the area of the rectangle. Be sure to include the appropriate units in your answer.

Possible answer: 9 × 10 = 90

The area of Mr. Hakim's patio is 90 square feet.

Student page 139

probing for understanding

For problems 1–3, call on students to sketch a diagram for the problem on the board or chart paper. Then have students share their solutions for all parts of the problem.

In your discussions of problems 1–3, make sure students are clear about the labels they use in their answers to the areas. Ask questions such as the following about each problem:

- Can you restate this problem in your own words?
- Where do you see the quantities given in the problem in your sketch?
- Where do we see the quantities in your equation?
- Who agrees with this solution? Who did something different?
- Did anyone try a method that did not work? Tell us what you tried.

In discussing problem 4, ask all students to share their ideas for problem situations involving area. Then choose one of the contexts suggested and ask that student to read the problem situation and the solution to the problem.

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- What was most challenging about writing a word problem involving area?
- What helped you choose a final idea?

teaching strategies

Make sure students are being precise in the manner in which they express their final answers. In particular, press students to be clear about the labels that should accompany the final answer to each problem. The answers should be in square measures, such as square inches and square feet. Observe whether students pay attention to this detail on their own or have to be prompted for this level of precision.

scaffolding for success

It is a good idea to draw each word problem rectangle on the board, label it, and point to its attributes as you discuss the problems with students.

reflection

Writing a Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

reflection

Finding the area of a rectangle can be useful because …

Student page 140

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Relating Area and Perimeter 27

Mathematical Goals

- Understands that rectangles with the same areas do not necessarily have the same perimeters
- Understands that rectangles with the same perimeters do not necessarily have the same areas

Misconceptions Addressed

- Does not know how to find perimeter
- Does not know how to find area
- Confuses area and perimeter

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD27-1 through MD27-5
✓ Chart paper

Optional Materials

✓ Grid paper

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD27-1 through MD27-5. During today's show me, students will find a rectangle with a given perimeter or area. Have students answer the following questions on their response boards.

- Show me a rectangle with …
  - … a perimeter of 10 units.

    Answer: rectangles with dimensions: 1 × 4, 4 × 1, 2 × 3, or 3 × 2
  - … an area of 10 square units.

    Answer: rectangles with dimensions: 1 × 10, 10 × 1, 2 × 5, or 5 × 2
  - … a perimeter of 8 units.

    Answer: rectangles with dimensions: 1 × 3, 3 × 1, 2 × 2
  - … an area of 8 square units.

    Answer: rectangles with dimensions: 1 × 8, 8 × 1, 2 × 4, or 4 × 2
  - … a perimeter of 12 units.

    Answer: rectangles with dimensions: 1 × 5, 5 × 1, 2 × 4, 4 × 2, or 3 × 3

To finish, have students complete the show me problem.

Relating Area and Perimeter 27

show me

Show me a rectangle with an area of 9 square units.

Possible answers:
a rectangle with dimensions 1 unit × 9 units, 9 units × 1 unit,
or 3 units × 3 units

Student page 141

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setting the direction

Ask a student to read the setting the direction problem aloud.

Instruct students to work solo to complete the setting the direction problem.

- How many rectangles did you find with a perimeter of 20 units? Tell me the different rectangles you found.

  Answer: Students could have found 9 possible rectangles that had a perimeter of 20 units: 1 × 9, 9 × 1, 2 × 8, 8 × 2, 3 × 7, 7 × 3, 4 × 6, 6 × 4, and 5 × 5.

setting the direction

Using only whole-number dimensions, find all the rectangles with a perimeter of 20 units. Sketch and label them below.

Student page 141

Make sure that students have included the rectangle that is also a square (the 5 × 5 rectangle).

- Which of the rectangles has the greatest area?

  Answer: The 5 × 5 rectangle will have the greatest area of 25 square units.

- Why do you think the square has the greatest area?

  Answer: Answers will vary. Students may say that the square has more space inside than the other rectangles, so the area should be greatest.

- Which of the rectangles has the least area?

  Answer: The 1 × 9 or 9 × 1 rectangle has the least area of 9 square units.

- Why do you think the 1 × 9 and the 9 × 1 rectangle have the least area?

  Answer: Answers will vary. Students may say that the longer, skinnier rectangle has less space inside, so the area will be the least.

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work time

Read the four problems aloud as students follow along.

Explain that students will work solo on problems 1 and 2 for a while and then they will finish all problems doing partner work.

After a few minutes of solo work, ask students to continue working on the task with a partner. Have them quietly discuss their solutions with their partner.

scaffolding for success

Pair students who need some help with students who can peer tutor them on the problems.

Spend some time conferencing with one or more students individually.

ERROR

As students work, watch for students who conclude that all shapes with a given perimeter have the same area or that all shapes with a given area have the same perimeter.

work time

1. Mr. Samson is building a rectangular pen for his dog. He has 24 yards of fencing.

   a. Complete the table to find all the possible pens that Mr. Samson can build using all the fencing. (Use whole-number dimensions.) Find the area of each pen.

Rectangle	Dimensions	Area
	1 yd × 11 yd or 11 yd × 1 yd	11 square yards
	2 yd × 10 yd or 10 yd × 2 yd	20 square yards
	3 yd × 9 yd or 9 yd × 3 yd	27 square yards
	4 yd × 8 yd or 8 yd × 4 yd	32 square yards
	5 yd × 7 yd or 7 yd × 5 yd	35 square yards
	6 yd × 6 yd	36 square yards

   b. Which pen has the greatest area?

   The pen that is 6 yards by 6 yards has the greatest area.

   c. Which pen has the longest length?

   The pen that is 1 yard by 11 yards has the longest length.

   d. Which pen would you advise Mr. Samson to build? Explain your answer.

   Answers will vary.

2. Mrs. Chi is making a quilt using 36 squares that are each 1 foot in length.

   a. Complete the table to find all the possible quilts that Mrs. Chi can make. Find the perimeter of each quilt.

Rectangle	Dimensions	Perimeter
	1 ft × 36 ft or 36 ft × 1 ft	74 feet
	2 ft by 18 ft or 18 ft by 2 ft	40 feet
	3 ft by 12 ft or 12 ft by 3 ft	30 feet
	4 ft × 9 ft or 9 ft × 4 ft	26 feet
	6 ft × 6 ft	24 feet

Student pages 142–143

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english language learners

Providing students with materials and giving them ample time to work on problems is especially important for ELLs. Providing grid paper to sketch the shapes helps strengthen the mathematical connections students make and also helps them make sense of more difficult abstract concepts.

scaffolding for success

To assist students with reading and content, you might need to guide them through each part of problems 1 through 4 or have their partners guide them through.

b. Mrs. Chi wants to sew ribbon around the edge of the quilt. Which quilt in the table needs the most ribbon? How much ribbon does Mrs. Chi need for this quilt?

The quilt that is 1 feet by 36 feet requires the most ribbon.
Mrs. Chi needs 74 feet of ribbon.

c. Which quilt in the table needs the least ribbon? How much ribbon does Mrs. Chi need for this quilt?

The quilt that is 6 feet by 6 feet needs the least ribbon.
Mrs. Chi needs 24 feet of ribbon.

d. There are 3 yards of ribbon in a package. If Mrs. Chi buys 3 packages of ribbon, which of the quilts in the table can she make? (1 yard = 3 feet)

3 yd × 3 ft = 9 ft
There is 9 feet of ribbon in a package.
9 ft × 3 = 27 ft
There is 27 feet of ribbon in 3 packages.
Mrs. Chi can make the quilt that is 4 feet by 9 feet or the quilt that is 6 feet by 6 feet with 3 packages of ribbon.

3. Look back at problem 1. What can you say about the areas of rectangles that have different dimensions but the same perimeter?

Answers will vary.
Responses should include the fact that areas increase as the lengths and the widths approach the same measure. A rectangle that is a square will have the greatest area.

4. Look back at problem 2. What can you say about the perimeters of rectangles that have different dimensions but the same area?

Answers will vary.
Responses should include the fact that perimeters decrease as the lengths and the widths approach the same measure.
A rectangle that is a square will have the least perimeter.

Student pages 144–145

probing for understanding

For problem 1, have each student share the dimensions of one of the rectangles they sketched and its area. Record the dimensions and area in an organized list on chart paper for the group to see.

- What did you notice about the areas of rectangles that have the same perimeters?

  Answer: Answers will vary. Students may observe the following:

  - Areas are different even if perimeters are the same.
  - The "squarer" the rectangle, the greater the area.
  - Rectangles that are the same but just rotated have the same area.
  - The square with a perimeter of 24 yards has the greatest area.

For problem 2, have each student share the dimensions of one of the rectangles they sketched and its perimeter. Record the dimensions and perimeter in an organized list on chart paper for the group to see.

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- What did you notice about the perimeters of rectangles that have the same areas?

  Answer: Answers will vary. Students may observe the following:

  - Perimeters are different even if areas are the same.
  - The "squarer" the rectangle, the less the perimeter.
  - Rectangles that are the same but just rotated have the same perimeter.
  - The square with an area of 36 square yards has the least perimeter.

teaching strategies

One of the key "big ideas" in mathematics is generalization. As a mathematical structure, generalization pushes the problem solver to examine a particular problem and make generalizations to similar problems using reasoning. Students attend to this practice in this lesson.

reflection

Writing a Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

reflection

One new thing I learned about area and perimeter is …

Student page 145

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Area and Perimeter Word Problems 28

Mathematical Goals

- Recognizes perimeter situations
- Recognizes area situations
- Represents measurement situations using visual representations
- Writes word problems involving area and perimeter

Misconceptions Addressed

- Does not know how to find perimeter
- Does not know how to find area
- Confuses area and perimeter

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD28-1 through MD28-5
✓ Chart paper

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD28-1 through MD28-5. During today's show me, students will find the value of an expression with more than one operation. Have students answer the following questions on their response boards.

- Show me the solution.
  - $(2 \times 3) + 12$

    Answer: 18
  - $(4 \times 5) + 15$

    Answer: 35
  - $(3 \times 6) + 16$

    Answer: 34
  - $(3 \times 4) + 13$

    Answer: 25
  - $(2 \times 5) + 17$

    Answer: 27

To finish, have students complete the show me problem.

Area and Perimeter Word Problems 28

show me

Show me the solution to: $(2 \times 4) + 15$ = 23

Student page 147

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setting the direction

Read the instructions for setting the direction aloud to students. Tell students that they will look at Josh's work and decide whether he is correct. Students must explain their answers.

Instruct students to work solo to complete the setting the direction problems.

After a few minutes, have students compare their answers with their partners' answers.

For each problem, ask questions such as the following:

- Is the problem a perimeter or area problem? How do you know?
- What makes a problem a perimeter problem?
- What makes a problem an area problem?

Have students brainstorm different contexts in which area situations occur and in which perimeter situations occur. Write students' ideas on chart paper.

Have students edit their answers to the setting the direction problems as needed.

setting the direction

For each problem below, Josh determined whether he needed to find the perimeter or the area to solve each problem. He circled his answer. Is Josh correct? Explain your answer.

Malaya wants to put trim around the edge of a square pillow. The pillow measures 18 inches by 18 inches. How much trim does she need?

area (perimeter)

Is Josh correct?

Josh is correct. To find the amount of trim Malaya needs, you need to find the perimeter of the pillow.

A baseball diamond is a square formed by home plate, first base, second base, and third base. The distance between bases is 90 feet. If a player hits a home run and runs around all the bases, how far does the player run?

area (perimeter)

Is Josh correct?

Josh is correct. To find the distance a player runs, you need to find the perimeter of the square formed by the bases.

Amir has a strawberry bed that measures 8 feet by 12 feet. He has 100 square feet of mulch. Does he have enough mulch to cover his strawberry bed?

area (perimeter)

Is Josh correct?

Josh is incorrect. To find out if there is enough mulch, you need to find the area of the strawberry bed.

Student pages 147–148

english language learners

Some ELLs may be uncomfortable speaking in front of the group. Be prepared to assist them with the language and allow them to use gestures and diagrams.

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work time

Explain to students that they will be writing and solving word problems related to the area and perimeter of rectangles. Have a student read the instructions for problems 1–3 aloud.

For each word problem, students will determine whether they need to calculate the area or the perimeter in order to answer the question. Students will circle "Area" or "Perimeter" once they decide. Then, students will calculate the solution to the word problem.

Next, using the same context given in the word problem, students will write their own related word problem that involves the measurement (area or perimeter) not found in the original problem. Students will provide a solution to their written problem.

Students will complete all parts of the activity with their partner. Make sure students provide a complete solution to each problem.

scaffolding for success

If students seem unsure of the task, model the steps for solving the problem before asking them to do the problems themselves.

work time

Work with your partner.

- For each of the following word problems, decide whether you need to find the area or perimeter. Circle "area" or "perimeter" once you decide.
- Then solve the word problem.
- Next, write a new word problem using the same situation and dimensions given in the original problem. If the original problem was about finding area, write a problem about finding perimeter. If the original problem was about finding perimeter, write a problem about finding area. Provide a solution for each of your problems.

1. Mrs. Chi needs to buy new carpet for a room. The dimensions of the room are 10 feet by 7 feet. How much carpeting does Mrs. Chi need to buy?

a. area or perimeter (circle one)

b. Solution:

$10 \times 7 = 70$

Mrs. Chi needs 70 square feet of carpeting.

c. New word problem and solution: Possible answer: Mrs. Chi wants to put a decorative strip of wallpaper on the walls of a room. The dimensions of the room are 10 feet by 7 feet. How much wallpaper will she need?

$2(10) + 2(7) = 20 + 14 = 34.$

Mrs. Chi will need 34 feet of wallpaper.

2. Tamika wants to put a ribbon border around her picture of her two best friends. If the picture is 5 inches by 7 inches, what length of ribbon does she need?

a. area or perimeter (circle one)

b. Solution: $2(5) + 2(7) = 10 + 14 = 24.$

Tamika needs 24 inches of ribbon.

c. New word problem and solution: The glass in Tamika's picture frame broke, and she needs to replace it. The frame is 5 inches by 7 inches. What is the area of the glass that she needs?

$5 \times 7 = 35$

Tamika needs 35 square inches of glass.

Student pages 148–149

english language learners

Some word problems may be problematic for ELLs. If students have difficulty, try to assess whether the problem for each student is the language or the math. Simple drawings, pictures, gestures, and demonstrations can help students understand what is being asked in some of the word problems.

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Do not expect students to finish all problems. Students are being asked to make sense of problems and persevere in solving them. Stop the work to give enough time for probing for understanding.

ERROR

Watch for students who confuse area and perimeter situations. These students may need more practice identifying situations in word problems. Encourage students to sketch diagrams of the situation if they are unsure as to whether it is an area or perimeter situation.

3. Anthony helped his dad cover one of the basement walls with paneling. The wall measures 20 feet by 8 feet. How big is the piece of paneling?

a. area or perimeter (circle one)

b. Solution: 20 × 8 = 160.
The piece of paneling is 160 square feet.

c. New word problem and solution:
Possible answer:
Anthony's dad put a new piece of paneling on one of the basement walls. Now he wants to put a wood border around the wall so the edges of the paneling will not show. The wall measures 20 feet by 8 feet. How many feet of the wood border does Anthony's dad need?
2(20) + 2(8) = 40 + 16 = 56.
Anthony's dad needs 56 feet of the wood border.

Student page 150

probing for understanding

Ask students questions.

- Was problem 1 an area or perimeter situation?
  Answer: area
- How do you know?
  Answer: Carpet covers an area of the floor; so to find how much carpet is needed, we will find the area.
- What related perimeter problem did you write for problem 1?
  Answer: Answers will vary. Possible answer: If you need to add trim on the edge of the carpeting that measures 10 feet by 7 feet, how much trim will you need? Solution: 34 feet
- How do you know your problem is a perimeter problem?
  Answer: Answers will vary. Possible answer: We are finding the length around the edge of the carpet, so it is perimeter.

Follow the same process to examine each of the other problems.

As time allows, have as many students share their ideas for alternative situations in each scenario.

english language learners

Encourage students to share their word problems. This is good practice for students who are working on their English skills.

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reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished. Have students respond to the reflection prompt.

reflection

One way I can figure out if a word problem involves area or perimeter is …

Student page 150

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The Distributive Property and Area 29

Mathematical Goals

- Connects the distributive property to the idea of breaking a rectangle into two smaller rectangles with the same number of rows
- Finds the area of a large rectangle by dividing the rectangle into two smaller rectangles and finding the area of each smaller rectangle and adding the areas together

Misconceptions Addressed

ERROR

- Does not know how to find area
- Confuses area and perimeter

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD29-1 through MD29-5
✓ Chart paper

Focus on Language

Model the use of this word and encourage students to use it throughout the lesson. Consider displaying the word so students can see it as they work.

- distributive property

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD29-1 through MD29-5. During today's show me, students will add two 2-digit numbers in their head. Have students answer the following questions on their response boards.

- Calculate the answer in your head.
  - 37 + 45

    Answer: 82
  - 53 + 19

    Answer: 72
  - 25 + 38

    Answer: 63
  - 15 + 66

    Answer: 81
  - 36 + 47

    Answer: 83

To finish, have students complete the show me problem.

The Distributive Property and Area 29

show me

Calculate the answer in your head: 19 + 25 = 44

Student page 151

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setting the direction

Tell students they are going to learn how the distributive property relates to area.

Read the setting the direction aloud to students.

Have students complete the setting the direction problems independently.

Once students are done, call on a variety of students to share their answers. Make sure students let you know if they disagree with an answer. Have students explain how they found their answers and then come to a final resolution at the end of the group discussion.

setting the direction

example

Anna says that she can calculate 3 × (4 + 5) in two ways.

3 × (4 + 5) = 3 × 9 = 27

3 × (4 + 5) = (3 × 4) + (3 × 5) = 12 + 15 = 27

Anna uses the distributive property in the second calculation.

Fill in the blanks below. Think about Anna's example above to help you.

(9 × 3) + (9 × 10) = 27 + 90 = 117

9 × (3 + 10) = 9 × 13 = 117

(3 × 5) + (3 × 6) = 15 + 18 = 33

8 × (2 + 8) = 8 × 10 = 80

Answers will vary. Possible answers:

Solve 6 × 10 in four different ways. 6 × (5 + 5) = 30 + 30 = 60

6 × (2 + 8) = 12 + 48 = 60

6 × (6 + 4) = 36 + 24 = 60

6 × (1 + 9) = 6 + 54 = 60

Student page 151

english language learners

Before you begin, have ELLs say in their own words what the distributive property is and display the term *distributive property* along with examples in the classroom.

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work time

Ask one student to read problem 1 aloud. Ask a different student to read problem 2.

Tell students that they will begin problem 1 doing solo work. Then after a few minutes, they will turn to their partner and complete both problems.

Have students work in pairs to provide support for each other "seeing" an array as two smaller arrays and then representing those smaller arrays with an equation using the distributive property. This can be quite challenging for some students. While students work, you can provide additional support for pairs who are experiencing difficulties.

ERROR

Watch for students who do not write the correct multiplication equation to find the area of a rectangle. Remind students that area is *length* × *width*. Have students count the number of square units in a row and the number of square units in a column to find the length and width.

work time

1. a. Determine the dimensions of Rectangles A and B.

Rectangle A Rectangle B

Rectangle A is 7 units by 6 units or 6 units by 7 units.

Rectangle B is 3 units by 6 units or 6 units by 3 units.

b. Find the area of Rectangle A and the area of Rectangle B.

Area of Rectangle A = 42 square units

Area of Rectangle B = 18 square units

c. Compute the sum of the areas of Rectangle A and Rectangle B.

Area of Rectangle A + Area of Rectangle B = 42 + 18 = 60 square units

d. Suppose Rectangle A and Rectangle B are combined into one rectangle, called Rectangle C. Find the dimensions of Rectangle C.

Rectangle C

Rectangle C is 10 units by 6 units or 6 units by 10 units.

e. Find the area of Rectangle C.

Area of Rectangle C = 60 square units

f. Explain how the work you did in parts a–e relates to the distributive property.

Possible answer:
Rectangle C has the same length as Rectangles A and B, but its width is the sum of the widths of Rectangles A and B. Its area is the same as the sum of the areas of Rectangles A and B. This can be shown by the distributive property:
6 × (7 + 3) = (6 × 7) + (6 × 3) = 42 + 18 = 60.

2. a. Find the area of the following rectangle.

3 × 7 = 21 or 7 × 3 = 21
The area is 21 square units.

b. Break the rectangle in part a into two smaller rectangles. Sketch the two rectangles. Find the area of each rectangle and the sum of their areas. Then write an equation using the distributive property to show how the combined area of the two smaller rectangles is equal to the area of the original rectangle.

1st rectangle: 4 × 3 = 12
The area is 12 square units.
2nd rectangle: 3 × 3 = 9 The area is 9 square units.
12 + 9 = 21 The sum of the areas is 21 square units.
3 × (4 + 3) = (3 × 4) + (3 × 3) = 12 + 9 = 21

Student pages 152–153

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scaffolding for success

Some students may have difficulty beginning problem 1 working alone. You may need to help them get started.

c. Break up the rectangle in part a in a different way. Find the area of each rectangle and calculate the sum of the two areas. Then write an equation using the distributive property to show how the combined area of the two smaller rectangles is the same as the area of the original rectangle.

1st rectangle: 2 × 3 = 6
The area is 6 square units.
2nd rectangle: 5 × 3 = 15 The area is 15 square units.
6 + 15 = 21 The sum of the areas is 21 square units.
3 × (2 + 5) = (3 × 2) + (3 × 5) = 6 + 15 = 21

probing for understanding

Student page 154

Review problem 1 with students. Call on individual students to show their answers.

- Is there another way to break Rectangle C into two smaller rectangles? Show us.
- What is the area of your two smaller rectangles?
- What is the sum of their areas?
- Is the sum of their areas the same as the area of Rectangle C?

For problem 2, focus discussion on the mathematical practice of looking for and making use of structure. Note students' recognition of the structure of their smaller rectangles and equations: all their ways of dividing the larger rectangle can be represented as the sum of two multiplication problems (the distributive property).

Have student pairs share their smaller rectangles and equations. Record their responses on chart paper.

Ask questions to elicit the structure of the distributive property:

- Where in your equations do you see the number of rows in the first smaller rectangle?
- Where in your equations do you see the number of squares in each row in the first smaller rectangle?
- Where in your equations do you see the number of rows in the second smaller rectangle?
- Where in your equations do you see the number of squares in each row in the second smaller rectangle?
- How are all of your rectangles and equations similar?
- What are we doing when we apply the distributive property in an area situation?

  Answer: We break a large rectangle into two smaller rectangles. The area of the large rectangle is equal to the sum of the areas of the two smaller rectangles.

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reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished. Have students respond to the reflection prompt.

reflection

It is helpful to be able to find the area of a rectangle in more than one way because …

Student page 154

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Using the Distributive Property

30

Mathematical Goals

- Use arrays to represent the distributive property
- Write an equation to match a visual representation

Misconceptions Addressed

ERROR

- Does not recognize or misapplies the distributive property
- Is unable to transfer between different representations of the same operation

Materials Needed

- ✓ Student pages and markers
- ✓ Response boards
- ✓ Show Me Cards MD30-1 through MD30-6
- ✓ Prepare grid paper strips; 13 strips with 7 squares in each strip
- ✓ Something long and thin like yarn or heavy string to use as a visual separator (approximately 10 inches long)
- ✓ Scissors
- ✓ Chart paper

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work.

- add
- distributive property
- plus
- sum

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards **MD30-1** through **MD30-6**. During today's show me, students will answer questions involving division by 3. Have students answer the following questions on their response boards.

- **MD30-1** How much is 0 divided by 3? Show me your answer as an equation.
- **MD30-2** One is equal to what number divided by 3?
- **MD30-3** Twelve equals 3 times what number?
- **MD30-4** Twenty-seven divided by 3 equals what number?
- **MD30-5** Eighteen equals 3 times what number?
- **MD30-6** How much is 21 divided by 3?

To finish, have students complete the show me problem.

Using the Distributive Property 30

show me

Write all the division facts you can think of for the number 24.

24 ÷ 1 = 24	24 ÷ 6 = 4
24 ÷ 2 = 12	24 ÷ 8 = 3
24 ÷ 3 = 8	24 ÷ 12 = 2
24 ÷ 4 = 6	24 ÷ 24 = 1

Student page 155

work time

Presenting the Task

Tell students they are going to be learning how they can use the distributive property to find solutions to problems. They will be making arrays and using a number line.

Model the use of a rectangular array as a visual representation for multiplication.

Read problem 1a. Tell students you are going to represent the fruit trees in Mr. Samson's orchard with grid paper. Organize students so that they have a clear view of your array. Lay out each strip of 7 squares.

scaffolding for success

Problem 1a has a lot of steps in it. You may need to help students break this down into smaller steps.

work time

1. Mr. Samson is planning to plant 13 rows of trees with 7 trees in each row. Some rows will have apple trees. The rest will have pear trees.

   a. Choose one way Mr. Samson could divide the rows into apple trees and pear trees. Show your way in the array below. Use A for each apple tree and P for each pear tree. Possible answers:

A	A	A	A	A	A	A
A	A	A	A	A	A	A
P	P	P	P	P	P	P
P	P	P	P	P	P	P
P	P	P	P	P	P	P
P	P	P	P	P	P	P
P	P	P	P	P	P	P
P	P	P	P	P	P	P
P	P	P	P	P	P	P
P	P	P	P	P	P	P
P	P	P	P	P	P	P
P	P	P	P	P	P	P
P	P	P	P	P	P	P

   b. Write an equation that uses the distributive property to match your divided array.

   (2 + 11) × 7

   c. Write the equation that shows the two products that you will add together.

   (2 × 7) + (11 × 7) = 91

   d. Write the equation with the sum of the two products.

   14 + 77 = 91

Student page 156

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- What do the 13 strips of grid paper represent?
  Answer: The 13 rows of fruit trees.
- What do the 7 squares on each strip of grid paper represent?
  Answer: The 7 fruit trees in each row.

Ask students to suggest how they can represent the array using multiplication.

Record on the board or a large sheet of paper: $13 \times 7 =$

Now place the yarn or string separator across the array between the sixth and seventh rows.

Explain that you are now dividing the array into two sets.

Point to the first set and ask:

- How many rows do we now have in this part of the array?
  Answer: We now have 6.
- How many trees are in each row?
  Answer: We still have 7 trees in each row.

Point to the second set and ask:

- How many rows do we now have in the second part of the array?
  Answer: We now have 7 rows.
- How many trees are in each row?
  Answer: We still have 7 trees in each row.
- Do we still have the same number of trees in our divided array as we had in our original array?
  Answer: Yes.
- What has changed in our divided array compared to our original array?
  Answer: We have separated the 13 rows into a set of 6 rows and a set of 7 rows.
- What has stayed the same in our divided array compared to our original array?
  Answer: The number of trees in each row has stayed the same.

Again point to the first set and then the second set as you say:

- 13 rows of 7 trees is equal to 6 rows of 7 trees (first set) plus 7 rows of 7 trees (second set).

Complete the equation on the board: $13 \times 7 = 6 \times 7 + 7 \times 7$

- Can you use number facts to find the products for the right side of the equation?

Write them on the board or the paper (under the previous equation): $13 \times 7 = 42 + 49$

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- What method can we use to find the sum of our two products?

Have several students share their methods.

Underneath the previous equations, write: $13 \times 7 = 91$

Before having students work on problem 1, be sure to ask:

- Is this the only way we could have divided the array?
- Can you give me an example of a different way we could have divided it?
- When you work on problem 1, I want you to divide it a different way.

scaffolding for success

Before starting problem 1, write the term "distributive property" on the board and have students define it in their own words. Then leave the term and some examples displayed for the lesson.

english language learners

You may need to pair new ELL students with partners who can help them with the language of the problem.

Have students work with partners to support each other in "seeing" an array as two separate sets and then representing those sets with an equation using the distributive property. This task can be quite challenging for some students.

While students work, you can move around and provide additional support for partners who are experiencing difficulties.

probing for understanding

Focus discussion on the mathematical practice of looking for and making use of structure. Note students' recognition of the structure of their divided arrays and equations: all their ways of dividing the orchard can be represented as the sum of two multiplication problems (the distributive property).

english language learners

It might help to keep the array for problem 1 displayed so that you can point to it as you ask questions and talk about problem 2.

Have partners share their divided arrays and equations. Record their responses on chart paper.

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Ask questions to elicit the structure of the distributive property:

- Where in your equations do you see the number of rows in the first part of your divided array?
- Where in your equations do you see the number of trees in each row in the first part of your divided array?
- Where in your equations do you see the number of rows in the second part of your divided array?
- Where in your equations do you see the number of trees in each row in the second part of your divided array?
- How are all of your arrays and equations similar?

work time

Presenting the Task

Have the students read problem 2 quietly to themselves, then tell you about the task in their own words.

Emphasize that this is quiet work time. Have grid paper and scissors ready for students to begin work. Encourage students to use the grid paper to model the situation.

After 1–3 minutes of solo work, ask students to join with partners to discuss and refine their answers.

2. There are 6 rows of seats in the school gymnasium.
In each row there are 15 seats.
How many seats are in the gymnasium?

a. Write an equation using the distributive property that you could use to find the number of seats.

Possible answer:
$6 \times 15 = (6 \times 10) + (6 \times 5)$

b. Solve the problem using your equation.

$6 \times 15 = (6 \times 10) + (6 \times 5)$
$= 60 + 30$
$= 90$

There are 90 seats in the school gymnasium.

Student page 157

probing for understanding

Allow students time to share their equations and explain their solutions.

Ask questions such as:

- <Name>, why did you divide your array the way that you did?
- What other way could you divide your array that would make it easier to find a solution?
- <Name>, where do you see the number of rows in your equation?
- <Name>, I noticed you divided your array in a particular way to begin with. Why didn't it work?

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Be sure to spend time on the first example given in the possible answers show above. This example divides the 15 using expanded form or place-value concepts. These concepts are the basis for the traditional procedure for multiplying larger numbers.

Remind students that first you multiply the 6 times the 5 ones, then you multiply the 6 times the 1 ten. Finally, you combine the two products.

In the traditional procedure, the place-value concepts are hidden—you will often hear students say "6 times 1" instead of "6 times 10" or "6 times 1 ten." You should be very explicit about place value as you work with students to multiply larger numbers.

Before closing the lesson, ask:

- What are we doing when we apply the distributive property in a multiplication situation?

  Answer: We spread our multiplication over two smaller numbers.

- How does using the distributive property help us find the solution in this multiplication situation?

  Answer: We can make the problem easier using the distributive property because we can multiply 6 by two smaller numbers, such as 5 and 10, instead of 15. Then we can add the two products.

reflection

Writing the Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

reflection

The distributive property can help make a multiplication problem easier because …

Student page 157

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Multiplying Multidigit Numbers 31

Mathematical Goals

- Multiplies multidigit whole numbers using strategies based on place value and properties of operations, in particular the distributive property

Misconceptions Addressed

ERROR

- Treats each place value of a whole number separately when multiplying and dividing
- Does not recognize or misapplies the distributive property

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD31-1 through MD31-5

Optional Materials

✓ Chart paper
✓ Base-10 blocks (9 thousands blocks, 10 hundreds blocks, 10 tens blocks, and 10 unit blocks per student)

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD31-1 through MD31-5. During today's show me, students will demonstrate their ability to add large numbers using number sense and mental computation. Have students answer the following questions on their response boards.

- Show me the sum.
  - 999,999 + 1

    Answer: 1,000,000
  - 590,090 + 1,010

    Answer: 591,100
  - 399,101 + 1,099

    Answer: 400,200
  - 250,500 + 50,500

    Answer: 301,000
  - 425,000 + 375,000

    Answer: 800,000

To finish, have students complete the show me problem.

Multiplying Multidigit Numbers 31

show me

Show me the number I would have to add to 500,000 to have 1,000,001.

500,001

Student page 159

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setting the direction

Read the setting direction problem to students. Explain that in this problem, only one row—the 4s row—is shown in each of three multiplication tables.

setting the direction

Fill in each of these multiplication tables.

×	1	2	3	4	5	6	7	8	9
4	4	8	12	16	20	24	28	32	36

×	10	20	30	40	50	60	70	80	90
4	40	80	120	160	200	240	280	320	360

×	100	200	300	400	500	600	700	800	900
4	400	800	1,200	1,600	2,000	2,400	2,800	3,200	3,600

Student page 159

- Do you recognize any of these multiplication tables? Where have you seen the first table?

  Answer: The first table is the basic multiplication facts table.

- Have you seen the second table?

  Answer: Students might have previously filled in this multiplication table, in which one set of factors is the multiples of 10 (10 through 90).

- So, the second table shows the products of multiplying one-digit numbers with multiples of 10. What does the third table show?

  Answer: The third table shows the products of multiplying one-digit numbers with multiples of 100 (100 through 900).

As you ask the following questions and take responses, have students record the products in the appropriate table on their student pages:

The first table

- What products will you write to fill in the first table, the 4s facts?

The second table

- What products will you write to fill in the second table—multiplying with multiplies of 10?

Write 4 × 20 on the board or chart paper. Then write the following and ask students to explain why these representations of 4 × 20 make sense:

4 groups of 2 tens 8 tens 80

- How many tens are in each product in the second table?

  Answer: 4 tens, 8 tens, 12 tens, and so on

- How is the second table like the first table? How is it different?

  Answer: The multiples of 4 are in both tables. In the second table, each product has a zero on the right, giving a result that is a number of tens. The products in the second table are 10 times as large as the (corresponding) products in the first table.

The third table

- What products will you write to fill in the third table—multiplying with multiplies of 100?

- How did you calculate the products when you multiplied with a multiple of 100?

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Elicit this strategy: Multiply the non-zero digits (find the "times fact"), and then to show that these are hundreds, shift the result to the left two places by writing two zeros at the end.

ERROR

As students fill in the tables, watch for students who add zeros without understanding why. Students who are unclear about multiplying with multiples of 10 and 100 may benefit by modeling using base-10 blocks. Plan to follow up with these students during partner work and conferencing.

Write 4 × 200 on the board or chart paper. Then write the following and ask students to explain why these representations of 4 × 200 make sense:

4 groups of 2 hundreds 8 hundreds 800 eight hundred

- How many hundreds are in each product in the third table?

  Answer: 4 hundreds, 8 hundreds, 12 hundreds, and so on

- How is the third table like the first table? How is it different?

  Answer: The multiples of 4 are in both tables. In the third table, the products have two zeros on the right, giving a result that is a number of hundreds. The products in the third table are 100 times as large as the products in the first table.

Explain that the four-digit products in the third multiplication table can be read two ways. For example, 2,400 can be read "two thousand, four hundred" and "twenty-four hundred." Have students take turns reading aloud the products both ways.

teaching strategies

In this lesson, it is important to include the interpretation of four-digit numbers as quantities of hundreds, such as "twenty-four hundred." Plan to help students clarify the two ways of reading four-digit numbers.

Have one student read the second problem aloud to the group.

Malaya said she could use the tables to find 4 × 136. She solved the problem this way.

4 × 100 = 400
4 × 30 = 120
4 × 6 = 24 400 + 120 + 24 = 520 + 24 = 544

Explain why Malaya's way works.

Possible answer:
Her way uses the distributive property. 4 × 136 is the same as 4(100 + 30 + 6) or (4 × 100) + (4 × 30) + (4 × 6).

Student page 159

- Who can explain how Malaya thought about 4 × 136?

  Answer: Take several responses so students can hear each other's explanations. Students are likely to explain correctly that Malaya "broke" (decomposed) 136 into 100, 30, and 6 and then multiplied each number by 4.

- How do you think that Malaya used the tables?

  Answer: She found the products of 4 × 100, 4 × 30, and 4 × 6 by looking in the appropriate tables: 4 × 100 in the bottom table, 4 × 30 in the middle table, and 4 × 6 in the top table. Then she added those (partial) products and the sum was 544.

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- Why does Malaya's way work?

  Answer: Her way uses the distributive property: Multiplying each of the three numbers (100, 30, and 6) and then adding the products gives the same result as multiplying the sum of the three numbers (136).

work time

Read problem 1 to students.

Tell students they will work with a partner on problem 1.

english language learners

Having students work in pairs provides students with opportunities to negotiate meaning, share ideas, and communicate with others during problem solving. This type of language-rich environment in which students are encouraged to work together and learn from each other is essential for ELL students' learning.

work time

1. Find 4 × 287 using these multiplication tables and Malaya's method.

×	1	2	3	4	5	6	7	8	9
4	4	8	12	16	20	24	28	32	36

×	10	20	30	40	50	60	70	80	90
4	40	80	120	160	200	240	280	320	360

×	100	200	300	400	500	600	700	800	900
4	400	800	1,200	1,600	2,000	2,400	2,800	3,200	3,600

Possible answer:
4 × 200 = 800
4 × 80 = 320
4 × 7 = 28 800 + 320 + 28 = 1,100 + 48 = 1,148

Student page 160

probing for understanding

Briefly have several students share their answers to problem 1. Record their strategies so other students can follow along and so the strategies are available for reference in the discussion.

ERROR

Problems 1–3 scaffold students' work using the distributive property to find partial products. Note any students who are unsure about the solution to problem 1. They will need extra support in problems 2 and 3.

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work time

Have a student read problem 2 aloud. Then, tell students that they will work on problem 2 doing partner work.

Observe students as they work. Note misconceptions revealed by students' written answers and conversations, and plan to incorporate these during probing for understanding.

2. Tran said he could find 2 × 348 without using the multiplication tables. Fill in the missing values in the equations below to complete Tran's work.

2 × 348 = 2 × (300 + 40 + 8)
= 600 + 80 + 16
= 696

Student page 160

scaffolding for success

Provide a safe setting for special needs students to ask questions about any misconceptions they may have. Help students break problems down by thinking aloud and asking questions that get them to think about the mathematics behind the problem. Thinking aloud can also be used to draw attention to common misconceptions. It is also important to model the way in which the language translates to mathematical terms and to a sequence of operations.

probing for understanding

Have a student share his answer to problem 2 as you record it on the board or chart paper.

Ask other students to agree with or challenge the answer and to justify their responses.

Highlight any misconceptions you observed during work time. You might propose an incorrect way of filling in the blanks and ask students to verify or challenge your work.

work time

Read problem 3 aloud to students.

Model "Ask Myself" questions using problem 3.

- As I am looking at this problem, I am asking myself questions to help me understand the problem:
  - Is this problem related to other problems in this lesson? I see that it is a multiplication problem and that a three-digit number is multiplied by a one-digit number.
  - How is this problem like the other problems in this lesson? All the problems in this lesson have involved multiplying a three-digit number by a one-digit number.
  - How is this problem different from the problems I have just done? First we had multiplication tables to use to help solve the problems. Then part of the computation was given to help us solve.

3. Solve this problem. Use the distributive property.

3 × 519

3 × 519 = 3 × (500 + 10 + 9)
= 1,500 + 30 + 27
= 1,557

Student page 161

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This time, there is no table and no partly worked computation. So, it looks like I will solve this problem on my own, from scratch.

- Can I use an approach I have used in this lesson? Yes. I could make multiplication tables, or I could do the computation without the tables.
- Would it help to look back at the other problems in the lesson? Yes. Problem 2 will help me remember how to write an equation using the distributive property. If I want to make tables, I can look at problem 1.

Tell students that they will work on problem 3 doing partner work.

As partners discuss the solution, monitor their thinking and explanations and make note of any points you feel you will need to focus on during probing for understanding.

Make note of any mathematical practices that you observe students using so that you can highlight these practices during the probing for understanding discussion. Students must look for and express regularity in repeated reasoning as they solve this problem using their understanding of place value and properties of operations.

probing for understanding

Have a student explain his or her computation strategy for 3×519 and record it on the board or chart paper. If another student has a different strategy, have that student present his work.

Lead a discussion of what students have learned about using place-value reasoning to multiply multidigit numbers by a single-digit number. Ask questions such as the following:

- Can you use what you have learned to multiply a two-digit number by a one-digit number?

  Answer: Yes, students can break any two-digit number into tens and ones, multiply those base-ten units, and add the products.

- Can you use what you have learned to find the product $3 \times 3{,}214$?

  Answer: Yes. The distributive property can be applied to solving this problem. Students may point out that this problem "is the same as finding 3×214 and adding $3 \times 3{,}000$."

- What property of operations are you using in the strategy you have used today?

  Answer: The distributive property.

- Will the method you have used today work for all problems in which you multiply a multidigit number by a single-digit number?

  Answer: This approach is a general method that will work for any problem of the type students have done today.

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teaching strategies

The computation strategy students have used in this lesson is the same as the standard algorithm, with the difference that students begin with the large values (they work from left to right). By dealing with the largest place value first, students achieve an estimate at the start of the computation. Another benefit of this approach is that it is a powerful mental math strategy.

- Could you explain the method you have used today to someone else?

  Answers will vary: Take this opportunity to reinforce the expectation that students should understand the computation strategies they use and use the strategies that they understand.

ERROR

This lesson introduces a partial-products strategy where students model the operation of the multiplier on the hundreds, the tens, and the ones values of the multiplicand. One strength of this approach is that students consider the value of the whole number rather than the individual digits. Building this understanding and habit of considering multidigit numbers when multiplying helps students avoid or undo the common error of treating each digit separately and disregarding its place value when multiplying.

reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished. Have students respond to the reflection prompt.

reflection

When I multiply a three-digit number, the distributive property helps me because …

Student page 161

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Multiplying by Two-Digit Numbers 32

Mathematical Goals

- Multiplies multidigit whole numbers using strategies based on place value and properties of operations, in particular the distributive property

Misconceptions Addressed

ERROR

- Treats each place value of a whole number separately when multiplying and dividing
- Does not recognize or misapplies the distributive property

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD32-1 through MD32-5

Optional Materials

✓ Chart paper

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD32-1 through MD32-5. During today's show me, students will use their understanding of the distributive property, their recall of multiplication facts, and their ability to multiply by 10 to solve equations involving the multiplication of three factors. Have students answer the following questions on their response boards.

- Show me how you would complete the equation.
  - $4 \times 9 \times 10 =$ ____ $\times$ ____ $=$ ____

    Answer: Accept $36 \times 10 = 360$, $4 \times 90 = 360$, or $9 \times 40 = 360$
  - $7 \times 8 \times 10 =$ ____ $\times$ ____ $=$ ____

    Answer: Accept $56 \times 10 = 560$, $7 \times 80 = 560$, or $8 \times 70 = 560$
  - $3 \times$ ____ $\times 10 = 210$

    Answer: $3 \times \mathbf{7} \times 10 = 210$
  - ____ $\times 6 \times 10 = 240$

    Answer: $\mathbf{4} \times 6 \times 10 = 240$
  - ____ $\times$ ____ $\times 10 = 490$

    Answer: $\mathbf{7} \times \mathbf{7} \times 10 = 490$

To finish, have students complete the show me problem.

Multiplying by Two-Digit Numbers 32

show me

Show me the number that makes this equation true.

$4 \times 8 \times$ 10 $= 320$

Student page 163

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setting the direction

Tell students that today they will use what they know about making sense of multiplication problems and using the distributive property as they multiply two two-digit numbers.

Read the setting the direction problem aloud.

Write the problem on the board or chart paper. Cover the 2 in 24 and ask the following question.

- How would you solve 4 × 13?

As you elicit different solution methods, ask students to make connections between the methods. For example, if an array diagram is presented, have students relate it to a solution using equations. Ask students to explain how the different representations show the use of the distributive property.

You might model this form of writing equations to show the strategy:

$$\begin{aligned} 4 \times 13 &= 4\,(10 + 3) \\ &= (4 \times 10) + (4 \times 3) \\ &= 40 + 12 \\ &= 52 \end{aligned}$$

Alternately, the same strategy can be recorded in this form:

$$\begin{aligned} 4 \times 10 &= 40 \\ 4 \times 3 &= 12 \\ 4 \times 13 &= 52 \end{aligned}$$

- So you know that 4 × 13 = 52. What else do you need to know to find this product: 24 × 13?

Elicit responses. Let students debate for a moment if different ideas are presented and then continue, using 20 × 13 as the next product needed.

setting the direction

Find one way to solve 24 × 13.

Student strategies will vary and may include equations, diagrams, and tables.

Possible answers:

$$\begin{aligned} 24 \times 13 &= 24(10 + 3) \\ &= (24 \times 10) + (24 \times 3) \\ &= 240 + 72 \\ &= 312 \end{aligned}$$

$$\begin{aligned} 24 \times 13 &= (20 + 4) \times 13 \\ &= (20 \times 13) + (4 \times 13) \\ &= 260 + 52 \\ &= 312 \end{aligned}$$

Student page 163

scaffolding for success

Making connections between different solutions for the same problem brings out the mathematics of the problem.

ERROR

Look for students whose response to the question "What else do we need to know?" is 2 × 13 rather than 20 × 13. These students may need more experiences decomposing and composing numbers, or they may need to correct the common error—treating each digit as a single digit—that can result from using the standard algorithm without understanding. Follow up with these students during partner work or conferencing time.

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- How would you solve 20 × 13? Let us hear a couple of ways.

Elicit more than one way and record all ways. For example:

$$\begin{aligned} 20 \times 13 &= 20(10 + 3) \\ &= (20 \times 10) + (20 \times 3) \\ &= 200 + 60 \\ &= 260 \end{aligned} \qquad \begin{aligned} 20 \times 13 &= 2 \times (10 \times 13) \\ &= 2 \times 130 \\ &= 260 \end{aligned}$$

Alternately, record this way:

$$\begin{aligned} 20 \times 10 &= 200 \\ 20 \times 3 &= 60 \\ 20 \times 13 &= 260 \end{aligned}$$

- Who can explain how we solved the whole problem, 24 × 13?

Have a student recap the process used. Invite the other students to offer suggestions that improve the clarity of the description of the process.

Help students generalize the method used to find 24 × 13. You might ask if students agree with this description: You broke one of the factors, 24, into the tens (20) and the ones (4). Then you multiplied the other factor, 13, by the tens (20) and the ones (4). Lastly, you added the resulting products.

teaching strategies

In this lesson students consider the place value of each digit in the factors rather than treat each digit as a single-digit numeral. Establishing the habit of considering the value of the digits when multiplying may take time. Continue to provide opportunities for students to use this approach.

work time

Have a student read problem 1 aloud.

Tell students they will do problem 1 working solo and then share their work with their partners. Remind them to be good listeners and ask questions when they are not clear about their partners' explanations.

work time

1. a. Solve 65 × 72. Use the distributive property. Write equations to show your thinking.

Possible answer:

$$\begin{aligned} 65 \times 72 &= 65(70 + 2) \\ &= 60(70 + 2) + 5(70 + 2) \\ &= (4{,}200 + 120) + (350 + 10) \\ &= 4{,}320 + 360 \\ &= 4{,}680 \end{aligned}$$

b. Compare your solution with your partner's solution. Did you work the problem the same way? Explain.

Answers will vary.

Student page 164

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probing for understanding

Have two or three students share their strategies for finding 65 × 72. Record the strategies, restating the student's explanation as you write and asking for clarification when you need it.

An example of one student's strategy recorded by the instructor is the following:

$$
\begin{aligned}
65 \times 72 &= (60 \times 72) + (5 \times 72) \\
&= (60 \times 70) + (60 \times 2) + (5 \times 70) + (5 \times 2) \\
&= (6 \times 10 \times 70) + (60 \times 2) + (5 \times 70) + (5 \times 2) \\
&= (6 \times 700) + (120) + (350) + (10) \\
&= (4{,}200) + (120) + (350) + (10) \\
&= 4{,}200 + 120 + 360 \\
&= 4{,}200 + 480 \\
&= 4{,}680
\end{aligned}
$$

teaching strategies

If hearing students' strategies and recording them is new to you, anticipate the strategies you might hear and the recording you would make. For the strategy recorded here, the student described her strategy using phrases such as, "First I thought…; then I said…; next I needed to…; then I just multiplied and got…; I added…"

english language learners

It may be helpful to color-code the equations to help students follow the explanation and to help them identify the use of the distributive property in the discussion that follows. For example, write (60 × 72) in red and (5 × 72) in black. In the following lines, use red for all parts of the equations related to (60 × 72) and black for parts of the equations related to (5 × 72).

- [Point to one strategy] Where is the distributive property used in this strategy?

Have students identify where the distributive property is used in one or two of the strategies that were shared. Invite students to point to the part of the equations they are speaking about.

Students should conclude that the distributive property was used several times.

- How does breaking apart each factor help you solve the problem?

Students are likely to respond correctly that breaking apart the factors makes it easier to compute.

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work time

Read both parts of problem 2 aloud as students follow along.

Tell students that they will work on both parts of the problem doing partner work.

Note students whose explanations show an understanding of Anthony's strategy so you can ask them to share their thinking during probing for understanding.

2. a. Anthony started to solve 42 × 59 this way.

42 × 59 = 59 × 42
59 × 42 = (60 × 42) – (1 × 42)

Try to understand his thinking. Write down what you understand.

He's thinking that multiplying by 60 would be easier.

And he can think about the problem as 59 × 42 and then think about 60 × 42.

Then, since 59 is 1 less than 60, he can find sixty 42s and subtract one 42.

b. Finish solving the problem Anthony's way.

42 × 59 = 59 × 42
59 × 42 = (60 × 42) – (1 × 42)
= 2,520 – 42
= 2,478

Student page 165

probing for understanding

Lead a discussion of problem 2 parts a and b.

Ask a student to describe what Anthony was thinking when he wrote the first equation, 42 × 59 = 59 × 42. Elicit the idea that Anthony used the commutative property: He understands that changing the order of the factors does not change the product.

Ask a different student to describe the thinking behind the second equation, 59 × 42 = (60 × 42) – (1 × 42), in Anthony's strategy shown on student page 165.

If some students had difficulty understanding Anthony's strategy, provide time for them to ask questions. Take this opportunity to encourage students to persist in tackling a problem. Make sure that the students are the audience for each other's questions and explanations, and encourage them to grapple with their questions and clearly explain their thinking.

Have students show how they filled in the blanks in problem 2, part b, to complete the solution.

If time allows, have students create problems that could be solved using Anthony's strategy and have other students tell how that would be done.

teaching strategies

Students who learned to use the standard multiplication algorithm without understanding may need more opportunities to develop missing conceptual knowledge. Consider presenting, on a regular basis, problems to solve using mental math (with recording to keep track), such as problem 2 in this lesson. Mental computation lends itself to using partial products and making sense of the numbers and operations as students work with them.

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reflection

Writing the Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

reflection

Here is a problem I can solve using Anthony's method:

Problem:

Solution:

Student page 166

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Relating Division to Multiplication 33

Mathematical Goals

- Divides numbers involving multidigit dividends using strategies based on place value and properties of operations, in particular the distributive property
- Applies understanding of place value and the relationship of division to multiplication to develop procedures to find quotients involving multidigit dividends

Misconceptions Addressed

ERROR

- Treats each place value of a whole number separately when dividing
- Does not recognize or misapplies the distributive property

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD33-1 through MD33-5

Optional Materials

✓ Chart paper

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- dividend *(dividendo)*
- divisor *(divisor)*
- partial quotient *(cociente parcial)*
- quotient *(cociente)*

english language learners

It is important to pre-teach or frontload new mathematical vocabulary before each lesson. Explicitly teach the division terms in this lesson so students can use understand them in discussions.

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD33-1 through MD33-5. During today's show me, students will find the missing factor in problems involving multiplying with multiples of 10. Have students answer the following questions on their response boards.

- Show me the missing number.
  - 25 × ____ = 250

    Answer: 10
  - 25 × ____ = 500

    Answer: 20
  - 10 × ____ = 150

    Answer: 15
  - 20 × ____ = 300

    Answer: 15
  - 32 × ____ = 640

    Answer: 20

To finish, have students complete the show me problem.

Relating Division to Multiplication 33

show me

Show me the missing number.

13 × 30 = 390

Student page 167

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setting the direction

Tell students that today they will explore a strategy for division.

Ask students to notice the three tables on their student pages.

Students might recognize that they have seen similar tables before.

Ask students to describe the tables:

- Only one row—the 6s row—is shown in each of three multiplication tables.
- The first table is the basic multiplication facts table.
- The second table shows the products of multiplying one-digit numbers with multiples of 10.
- The third table shows the products of multiplying one-digit numbers with multiples of 100.

setting the direction

Fill in each of these multiplication tables.

×	1	2	3	4	5	6	7	8	9
6	6	12	18	24	30	36	42	48	54

×	10	20	30	40	50	60	70	80	90
6	60	120	180	240	300	360	420	480	540

×	100	200	300	400	500	600	700	800	900
6	600	1,200	1,800	2,400	3,000	3,600	4,200	4,800	5,400

Maria solved 4,722 ÷ 6 by thinking, "How many 6s in 4,722?"
She used the tables to make groups of 6. She used each table once.
Try to understand Maria's work, and then fill in the blanks below.

700 x 6 = 4,200 — 4,722 − 4,200

80 x 6 = 480 — 522 − 480

7 x 6 = 42 — 42 − 42 = 0

I used 787 groups.

(700 + 80 + 7) × 6 = 4,200 + 480 + 42

Student page 167

Have students tell you the missing products as they fill in the tables.

Read aloud the problem below the tables about Maria's strategy for solving 4,722 ÷ 6.

scaffolding for success

Seeing easier problems of this type may help some students understand Maria's approach. Elicit suggestions for easier problems, such as:

Solve 24 ÷ 6 by thinking, how many 6s in 24.

Solve 70 ÷ 10 by thinking, how many 10s in 70.

Solve 250 ÷ 50 by thinking, how many 50s in 250.

Have students study Maria's work to respond to the following questions.

- Which table did Maria use to find 700 × 6 = 4,200?

  Answer: The multiples of 100 table

- Why did Maria make 700 groups of 6? Could she have made 800 groups of 6?

  Answer: Maria wants to know what number times 6 is close to 4,722 without exceeding it. She made 700 groups because that is 4,200—close to 4,722. She could not have made 800 groups because that would be 4,800: more than 4,722.

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teaching strategies

Remind students of the two interpretations of four-digit numbers they know: as quantities of hundreds, such as "forty-two hundred," and as a number of thousands and hundred, such as "four thousand, two hundred."

- Who can explain why Maria subtracted 4,200 from 4,722?

  Answer: Because 4,200 is less than 4,722, Maria knows she needs to make more groups of 6. When Maria subtracts, she finds out that she has 522 more to put into groups of 6.

- Maria has used the bottom table once. What table do you think she used to find the groups of 6 in 480?

  Answer: The middle table shows that 80×6 is 480.

Have students fill in the blank to show $80 \times 6 = 480$ and then write 480 in the running subtraction record at the right.

- How many groups of 6 has Anna made so far?

  Answer: First she made 700 groups. Then she made 80 groups. She has made 780 groups so far.

- When Maria subtracts 480, how many are left?

  Answer: 42

- What table shows how many 6s are in 42?

  Answer: The top table shows the multiplication fact $7 \times 6 = 42$.

- How will you fill in the blanks?

  Answer: $7 \times 6 = 42$; write 42 in the running subtraction record.

- Where does Maria's recording show the number of groups of 6 she made?

  Answer: The first factor in each equation shows the number of groups of 6: 700, 80, and 7. Adding those up, you get 787. That goes in the blank in her sentence "I used _____ groups."

- How should you complete the equation Maria wrote at the bottom of her work? Explain how you know.

  Answer: In the left side of the equation, Maria shows the number of groups of 6 (700, 80, and 7) she used for each part of 4,722 (4,200; 480; 42) she worked with. In the right side, she shows the product of each multiplication (the partial quotients). The equation is a shorter way of showing all of Maria's work above.

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work time

Have a student read aloud problems 1–3.

ERROR

Watch for students who have learned the standard algorithm for division and are having trouble thinking about an entire number (the dividend) and the place value of the digits in that number. The trouble stems from the fact that the standard algorithm frequently functions for students as a memorized procedure in which they treat the dividend as a set of individual digits considered one by one. Let students know that both ways work, and that knowing how to work in the way this lesson presents will build students' ability to work with large numbers in their head and their ability to estimate. You might ask these students just to estimate the answer to division problems like those in this lesson.

Tell students to work on all parts of problem 1 with a partner.

Tell students to work on problem 2 doing solo work and then check their answers with their partners.

Observe students as they work. Notice student thinking and any misconceptions you want to address in probing for understanding.

work time

1. a. Fill in each of these multiplication tables.

×	1	2	3	4	5	6	7	8	9
8	8	16	24	32	40	48	56	64	72

×	10	20	30	40	50	60	70	80	90
8	80	160	240	320	400	480	560	640	720

×	100	200	300	400	500	600	700	800	900
8	800	1,600	2,400	3,200	4,000	4,800	5,600	6,400	7,200

b. 7,896 ÷ 8
How many 8s in 7,896?

Use groups of 8 from the tables. Use the fewest steps possible.

900 × 8 = 7,200
80 × 8 = 640
7 × 8 = 56

7,896
−7,200
696
− 640
56
− 56
0

Total 987 groups.

c. Write an equation to show how you used groups of 8.

7,896 ÷ 8 = (900 + 80 + 7) × 8 = 7,200 + 640 + 56

2. a. 3,780 ÷ 5
How many 5s in 3,780? Try this problem without tables.

700 × 5 = 3,500
50 × 5 = 250
6 × 5 = 30

3,780
−3,500
280
− 250
30
− 30
0

Total 756 groups.

b. Write an equation to show how you used groups of 5.

3,780 ÷ 5 = (700 + 50 + 6) × 5 = 3,500 + 250 + 30

Student pages 168–169

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Tell students to write their explanations for problem 3 doing partner work.

Observe whether students are familiar with remainders in division problems.

If time is short, stop students before they finish problem 3 so there is time for the discussion in probing for understanding.

3. What is the same about the problem below and the one you just did? What is different?

3,781 ÷ 5

Possible answer:

What's the same is that 3,780 and 3,781 have the same number of 5s. What's different is that 3,781 has one more, and it's not enough to make another group of 5. It's just left over.

Student pages 169

probing for understanding

Discuss problems 1 and 2. Ask questions to review what students have learned and to understand their thinking. Make sure the discussion elicits these ideas:

- To solve the division problems, students thought about multiplication. They might have thought about looking for a missing, or unknown, factor. "What number times 5 equals 3,780?" or "The product is 3,780. One factor is 5. I want to find the other factor."
- Students divided *parts* of the total amount (the dividend) until they had divided the entire number. They then added up the results of those partial divisions (the partial quotients), giving the total results (the total quotient). In working this way, they were applying the distributive property.

Present a new problem represented in the following way, and check that students are familiar with this division symbol:

$5\overline{)4,320}$

Have students use Maria's method to answer questions such as the following. As students respond, create the recording shown at right.

- Will 1,000 groups of 5 work?
- How many hundreds times 5 can you make?
- How do we find out how many are left to group?
- How many groups can you make now?
- Who can tell me how many groups we made altogether?

$$
\begin{array}{rl}
5\overline{)4,320} & \\
-\ 4,000 & 800 \times 5 = 4,000 \\
\hline
320 & \\
-\ 300 & 60 \times 5 = 300 \\
\hline
20 & \\
-\ 20 & 4 \times 5 = 20 \\
\hline
 & 800 + 60 + 4 = 864
\end{array}
$$

teaching strategies

This look at $5\overline{)4,320}$ is introduced here simply to connect to students' awareness of the division symbol $\overline{)\quad}$. You should not teach the long division algorithm now. Do note if any students are familiar with the process used by Maria and recorded by you here.

Have one or two students share their explanations for problem 3. Since the answer is fairly straightforward, you might focus students' attention on the clarity of the explanations offered.

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reflection

Writing the Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

reflection

When I am dividing, thinking about multiplication helps because …

Student page 170

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Estimating When Dividing 34

Mathematical Goals

- Applies understanding of place value and the relationship of division to multiplication to develop procedures to find quotients involving multidigit dividends
- Assesses the reasonableness of answers using mental computation and estimation strategies, including rounding

Misconceptions Addressed

- Estimates incorrectly
- Treats each place value of a whole number separately when dividing

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD34-1 through MD34-5

Optional Materials

✓ Chart paper

Focus on Language

Model the use of this word and encourage students to use it throughout the lesson. Consider displaying the word so students can see it as they work. The Spanish cognate is in parentheses.

- per *(por)*

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD34-1 through MD34-5. During today's show me, students will demonstrate their ability to look at a number and think of it in terms of another base-ten unit, such as 3,654 as 36 hundreds, 365 tens, and so on. Have students answer the following questions on their response boards.

- Fill in the blanks to make the statement true:
  - There are _____ hundreds in 1,200.

    Answer: 12
  - There are _____ hundreds in 1,640.

    Answer: 16
  - There are _____ ones in 1,253.

    Answer: 1,253
  - There are _____ ten thousands in 790,000.

    Answer: 79
  - There are _____ hundreds in 65,200.

    Answer: 652

To finish, have students complete the show me problem.

Estimating When Dividing 34

show me

Show me the product.

13 × 300 = 3,900

Student page 171

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setting the direction

Begin by having students cover their student pages.

Read aloud the first part of the setting the direction problem, the situation.

Miguel's school recycled 3,417 cans in 32 weeks. Miguel said,

> "That's a little more than 100 cans per week."

Read that part of the problem again as you write on the board or chart paper:

Recycled: 3,417 cans 32 weeks

setting the direction

Miguel's school recycled 3,417 cans in 32 weeks. Miguel said,

"That's a little more than 100 cans per week."

Student page 171

english language learners

Students may not be familiar with the word *recycle*. Ask students to explain the meaning, and be prepared to support their explanations and clarify the meaning.

Ask a student to state in his own words what he knows about this problem. His statement might be as brief as, "The school recycled 3,417 cans in 32 weeks, and Miguel thinks that's more than 100 cans per week."

Ask students to share anything else they know about the problem. Ask the questions below to draw out ideas that students do not offer.

- At the end of 32 weeks, how many cans had the school recycled?

  Answer: 3,417

- Do you know from the problem how many cans the school recycled each of the 32 weeks?

  Answer: No. We do not know how many cans were recycled each week.

- What does "100 cans per week" mean in this problem?

  Answer: A sample correct student explanation is: "If you think about all the cans and all the weeks and you spread out the cans evenly over all those weeks, then there would be a little more than 100 cans for each week. That is what it means. It is an average. It does not mean that the school recycled a little more than 100 cans every week."

scaffolding for success

Be aware that students may not be familiar with the term *per* as used in the context of this problem. The first questions provided here should help clarify the meaning for the purpose of solving this problem. You might want to discuss an example such as: "I read a 362 page book in 7 nights. Some nights I read for a long time, and some nights I fell asleep before I could read 10 pages. But I am pleased that I am reading about 50 pages per night."

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- What do you think? Do you agree with Miguel?

Take responses. Elicit the reasoning that 32 × 100 = 3,200, which is close to 3,417.

Ask students to find student page 171 and write why Miguel's estimate makes sense.

Explain why Miguel's estimate makes sense.

Possible answer:

Miguel's estimate makes sense because 32 × 100 = 3,200, which is close to 3,417.

Since 3,200 is a little less than 3,417, Miguel is right that the school recycled a little more than 100 cans per week.

Student page 171

teaching strategies

Presenting word problems is a good way to develop students' estimation skills. Problem situations prompt students to make sense of the numbers involved rather than apply an estimation procedure without thinking. Also, word problems can introduce or highlight situations in which estimation is actually used and in which an estimate is good enough, an exact answer is not needed.

work time

Read the directions to both parts a and b of problem 1 to students as they follow along.

- Their job is to look at each division problem and estimate whether the answer is less than 10, equal to 10, or more than 10.
- They should not use a calculator or do pencil-and-paper calculations. They should decide by estimating in their head.
- If they think the result of the division, the quotient, is less than 10, they should write the problem in the first column, headed "____ < 10."
- If they think the result of the division is equal to 10 (exactly 10), they should write the problem in the second column, headed "____ = 10."
- If they think the result of the division is more than 10, they should write the problem in the third column, headed "____ > 10."
- Finally, they should make up one division problem for each column of the table, and record their problems in the last row of the table.

Tell students to work on problem 1 doing solo work and then check their answers with their partners.

work time

1. a. In the table below, write each expression in the column that describes the result of the division. Do not use paper and pencil or a calculator. Just use estimation.

600 ÷ 60	4,999 ÷ 500	700 ÷ 50
756 ÷ 80	660 ÷ 60	5,000 ÷ 200
500 ÷ 50	700 ÷ 70	536 ÷ 60

_____ < 10	_____ = 10	_____ > 10
4,999 ÷ 500	600 ÷ 60	700 ÷ 50
756 ÷ 80	500 ÷ 50	660 ÷ 60
536 ÷ 60	700 ÷ 70	5,000 ÷ 200
Expressions will vary.	Expressions will vary.	Expressions will vary.

b. In the bottom row of the chart, write another expression for each column.

Answers will vary.

Student page 172

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ERROR

Look for students who have difficulty figuring out which column a division problem belongs in. They may need more work with multiplies of 10 and 100. Plan to follow up with these students. You might have them, for example, skip count by 500 on a number line marked in intervals of 500, and stop when they are as close to 4,900 as they can get. Ask how many jumps they made to get to 500, and have them write the equation: $10 \times 500 = 5{,}000$.

Tell students to work on problem 2 with their partners.

Observe students as they work. Notice student thinking and any misconceptions you want to address in probing for understanding.

Now is also a good time to observe students' comfort level with problem solving as well as their ability to persevere if the problem is not straightforward to them.

2. Estimate the answer. $3{,}510 \div 6$

Explain how you estimated.

Answers will vary. A strong estimate is 600. A strong estimation strategy is to think about $3{,}600 \div 6$. Students may explain that they arrived at 3,600 by looking for an "easy number" to change 3,510 to or by thinking about 6 times multiples of 100: 600; 1,200; 1,800; 2,400; 3,000; 3,600 (like using the 100s times multiplication table in previous lessons in the module).

Student pages 172

probing for understanding

Discuss problems 1 and 2. Ask questions to review what students have learned and to clarify their thinking.

- Let us talk about problem 1. How would you explain to someone else how to figure out which columns the division problems belong in?

Have students propose explanations. A sample line of reasoning is the following:

- Look at the divisor and the dividend. Check that the dividend has one more place than the divisor because then it is useful to think about the product of the divisor times 10.
- The divisor times 10 will give you a number with the same leading digit in the dividend—in the next place to the left.
- If the dividend has a leading digit less than the divisor's leading digit, the answer will be less than 10.
- If the dividend has a leading digit greater than the divisor's leading digit, the answer will be more than 10.
- If the dividend has the same leading digit followed by zeros, and no other digits, the dividend is equal to 10; it is exactly 10. (This case gives an exact answer.)

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- For the division problems in problem 1, is it easy to tell when the answer is more than 10? What would you say?

scaffolding for success

This second question is intentionally posed after the first question. It allows students to develop and consolidate new learning from hearing the discussion of the first question. Some students who might have held back if the second question had been asked first will now be able to participate in the discussion.

Answers will vary: They should include that the answer is more than 10 when the dividend is more than 10 times the divisor, and it is easy to tell when a number is 10 times another number.

- Let us hear your estimates for 3,510 ÷ 6.

Write on the board or chart paper the estimate or estimates students arrived at. Leave room to record students' estimation strategies below the estimates.

- How did you arrive at your estimate?

Ask two students with the same estimate to share their strategies. If the strategies are different, have students compare them. Students should verify, challenge, and justify the ideas offered.

If one or two other estimates were arrived at, ask the students who made those estimates to share their strategies. Again, have students respond to the strategies.

Make sure the estimate 600 has been considered.

- If students have not yet discussed whether 600 is an overestimate or an underestimate, ask, "Will the exact answer be more than 600 or less than 600?"

  Answer: The exact answer, the quotient, will be less than 600 because 3,510 is less than 3,600. Since there are exactly six hundred 6s in 3,600, there must be fewer than six hundred 6s in a number less than 3,600.

- I noticed that you changed the dividend, the number that was being divided—3,510. You did not change the divisor. Did Miguel use the same strategy?

  Answer: Yes, Miguel used the same strategy. Instead of thinking about 3,417, he thought about 3,200 when making his estimate. In making his estimate, Miguel did not change the divisor.

- Do you think you will use this strategy for other problems when you want to estimate?

  Answers will vary: Take this opportunity to set the expectation that students are learning estimation strategies they should choose from as they carry out division calculations.

teaching strategies

In this problem, rounding the dividend to the nearest thousand or hundred produces numbers that are not very useful: 4,000 and 3,500. One way to help students achieve fluency with estimation is to present problems for which different strategies result in good estimates. The goal for students is to be familiar with a number of useful estimation strategies and to be able to select effective strategies to use.

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reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished. Have students respond to the reflection prompt.

reflection

One thing that helps me estimate when I am dividing is …

Student page 173

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Interpreting Remainders 35

Mathematical Goals

- Evaluates student work and identifies any errors in the interpretation of remainders in division situations

Misconceptions Addressed

- Does not deal with remainders in problem situations appropriately, including thinking remainders can be greater than divisors

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD35-1 through MD35-5

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- error (*error*)
- misconception

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD35-1 through MD35-5. During today's show me, students will find the missing number in a factor pair for a number. Have students answer the following questions on their response boards.

- Show me the missing number in the factor pair for …
- … 81: 9 and ____
  Answer: 9
- … 40: ____ and 8
  Answer: 5
- … 54: ____ and 9
  Answer: 6
- … 70: 7 and ____
  Answer: 10
- … 64: ____ and 8
  Answer: 8

To finish, have students complete the show me problem.

Interpreting Remainders 35

show me

Show me the missing number in the factor pair for 72: 8 and 9

Student page 175

setting the direction

Introducing the Lesson

Tell students that in this lesson, students will be asked to correct Anthony's homework problems. Students should mark whether each problem is correct or incorrect. If the problem is wrong, students should find the right answer and then explain to Anthony what he is doing wrong.

work time

Anthony turned in solutions for the following homework problems.

- Mark whether each problem is correct or incorrect. Use a colored pen, if possible.
- Correct the problems he got wrong.
- With your partner, write comments to Anthony explaining what he is doing wrong.

Student page 175

Explain that sometimes students make mistakes in their work. Sometimes these mistakes are careless errors, and sometimes they happen because students try to use methods that do not work or use something they learned that is not correct. These mistakes are called misconceptions.

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teaching strategies

Today's lesson has four problems. You will need to watch the time, allowing no more than 5 to 6 minutes per problem, including the discussion.

Ask a student to read problem 1 aloud, and then ask another student to explain it in her own words.

Have students work solo initially on this activity and then turn to their partners to explain and compare their solutions.

scaffolding for success

Partner work provides students with the opportunity to work cooperatively, in order to negotiate and share ideas and communicate with each other during problem solving. Such group-learning settings provide special needs students a chance to hear and use mathematical language while building their communication skills.

Name: Anthony

1. Malaya has a new kitten. The kitten eats 3 ounces of cat food each day. How many days can Malaya feed the kitten from a 1-pound bag of cat food?

Note: 1 pound is equal to 16 ounces.

15 ÷ 3 = 5; Malaya can feed the kitten for about 5 days.

Let d = the number of days.

d = 16 ÷ 3

d = 5 R1

Malaya can feed the kitten for 6 days.

One ounce is not enough to feed the kitten for another whole day. Malaya can only feed her kitten for 5 days, not 6.

Student page 176

ERROR

Look for students who do not correctly identify the errors in Anthony's work. In three of the four problems, Anthony handles the remainder incorrectly. Let students use manipulatives to act out the problem situations. Ask students to reason how to handle the remainder in each case depending on the problem situation.

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probing for understanding

Ask questions such as the following:

- Where do you see the 1-pound bag in Anthony's diagram? In his equation?

  Answer: In the number line, the 1-pound bag is shown by 16 ounces because there are 16 ounces in 1 pound. In the equation, the dividend 16 is the 1-pound bag because it is the total amount being divided.

- Where do you see the size of each serving in Anthony's diagram? In his equation?

  Answer: The size of a serving is 3 ounces. In the number line, each interval is a 3-ounce serving. In the equation, the divisor 3 is the amount of one serving.

- Where do you see the 5 days in Anthony's diagram? In his equation?

  Answer: In the number line, there are 5 intervals of 3. This represents the 5 days. In the equation, the whole-number quotient is the number of days.

- What does the remainder mean in this situation?

  Answer: The remainder means there is 1 ounce of cat food left.

- Does Anthony interpret the remainder correctly in this situation? Explain why or why not.

  Answer: No, Anthony rounds the remainder up to another day when there is not enough cat food for a sixth day.

work time

Have students read the problem silently to themselves and then check for understanding.

Have students work solo initially on this activity and then turn to their partners to explain whether Anthony solved the problem correctly.

english language learners

Encourage students to talk to one another as they work in pairs. Students must use mathematical language in order to develop it. Encourage the use of English without discouraging students from using their native languages.

2. The fifth-grade class is going canoeing. There are 53 students on the trip. Each canoe will hold at the most 3 students. How many canoes will the class need?

60 ÷ 3 = 20; about 20 canoes will be needed.

Let c = the number of canoes needed.

c = 53 ÷ 3

c = 17 R2

The class needs 18 canoes.

Anthony is correct.

Student page 177

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probing for understanding

Anthony solved the problem correctly. Focus the discussion on how Anthony handled the remainder. Ask questions such as the following:

- Why is the answer 18 canoes instead of 17 canoes?

  Answer: There are 2 students left over, so they will need a canoe as well.

- Why does an answer of 17 R 2 canoes not make sense?

  Answer: The situation is talking about number of canoes needed. It does not ask for the number of canoes that can be filled and the number of people left over.

- Why does an answer of 17 $\frac{2}{3}$ canoes not make sense?

  Answer: You cannot have a part of a canoe. A whole canoe is needed for the extra 2 students.

work time

Read the problem aloud as students follow along.

Have students work independently on the problem and then check in with their partners. Partners should be able to explain why they think Anthony's solution is correct or incorrect.

3. Miguel is wrapping packages. He has 76 feet of ribbon to make 8 bows. He wants to use all of the ribbon. How much ribbon does he have for each bow?

80 ÷ 8 = 10; Miguel has about 10 feet of ribbon for each bow.

Let f = the number of feet of ribbon for each bow.

f = 76 ÷ 8

f = 9 R4

Miguel has 9 feet 4 inches for each bow.

Anthony forgot that the units are in feet, so the remainder is also in feet. Since feet are commonly divided into fractional quantities (unlike people or canoes), he should divide the remainder 4 by 8 to get that each bow will use $9\frac{1}{2}$ feet or 9 feet 6 inches of ribbon.

Student page 178

probing for understanding

Focus most of the discussion for this problem on the remainder. Ask questions such as the following:

- Did Anthony interpret the remainder correctly? Explain how you know.

  Answer: No, Anthony thought the remainder represented 4 inches. There are 4 feet of ribbon left over.

- How can you represent the remainder accurately?

  Answer: You can use a fraction instead of a remainder. There are 4 feet left to divide among 8 bows. That is $\frac{1}{2}$ foot per bow, so Miguel has $9\frac{1}{2}$ feet for each bow.

- Does anyone else have another way to represent the remainder? Explain your thinking.

  Answer: You could also say that Miguel has 9 feet 6 inches for each bow since $\frac{1}{2}$ foot is equal to 6 inches.

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work time

Have a student read problem 4 aloud.

Again have students work solo and then compare their answer with their partners. Encourage students to use mathematical language when explaining whether Anthony solved the problem correctly.

4. The flower market is having a sale on tulips. What is the cost of each bunch of tulips?

Tulips
4 bunches for $26

$28 ÷ 4 = $7; each bunch is about $7.

Let c = the cost of each bunch of tulips.

c = $26 ÷ 4

c = $6 R2

c = $8

Each bunch of tulips cost $8.

Since dollars can be divided into fractional parts (unlike cars or canoes) and the remainder is $2.00, it should be divided among the 4 bunches.
So each bunch costs:
$6.00 + ($2.00 ÷ 4) = $6.00 + $0.50 = $6.50

Student page 179

probing for understanding

Ask questions that focus on the remainder.

- Did Anthony interpret the remainder correctly? Explain why or why not.

  Answer: No, Anthony misinterpreted the remainder. He added the remainder to the quotient.

- How should Anthony have interpreted the remainder? Explain your thinking.

  Answer: Anthony had $2 left over. He should have divided the amount among the cost of the 4 bunches. If you divide $2 by 4, you get $0.50 per bunch. So each bunch costs $6 + $0.50, or $6.50.

reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished. Have students respond to the reflection prompt.

reflection

To help Anthony interpret remainders, I would tell him …

Student page 179

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Repeating Shape Patterns 36

Mathematical Goals

- Generates repeating shape patterns that follow a given rule
- Identifies apparent features of the pattern that were not explicit in the rule itself

Misconceptions Addressed

ERROR

- Does not recognize a pattern, including a number pattern
- Does not understand how to use number patterns to solve problems

Materials Needed

✓ Student pages and markers
✓ Response boards
✓ Show Me Cards MD36-1 through MD36-5
✓ Scissors and glue (one per student)

Focus on Language

Model the use of these words and encourage students to use them throughout the lesson. Consider displaying the words so students can see them as they work. Spanish cognates are in parentheses.

- core
- design (*diseño*)
- element (*elemento*)
- pattern (*patrón*)
- repeating pattern
- rule

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show me

RESPONSE BOARDS

Begin the lesson by using Show Me Cards MD36-1 through MD36-5. During today's show me, students will identify the remainder of a division problem. This skill will help them name which element is in a certain position in a pattern. Have students answer the following questions on their response boards.

- Show me the remainder for …
- … 38 ÷ 3
  Answer: The remainder is 2.
- … 43 ÷ 2
  Answer: The remainder is 1.
- … 101 ÷ 3
  Answer: The remainder is 2.
- … 50 ÷ 2
  Answer: There is no remainder.
- … 999 ÷ 3
  Answer: There is no remainder.

To finish, have students complete the show me problem.

Repeating Shape Patterns 36

show me

Show me the remainder for 10 ÷ 3. The remainder is 1.

Student page 181

setting the direction

Tell students that today's lesson is about repeating patterns.

setting the direction

- Get the *Shapes for Patterns* page (page 185) and carefully cut out all the shapes.
- Use the given rule to make a pattern with the shapes.
- Once you believe your pattern is correct, ask your teacher to check it. Your teacher will then tell you if you can glue the shapes down.

Student page 181

- Why do you think we study patterns?
  Answer: Answers will vary. Students may say that we study patterns so that we can make interesting designs. However, make it clear to students that beyond creating things that are pretty or fun, patterns help us predict and plan.

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- Can anyone think of a pattern that repeats?

  Answer: Answers will vary. Students may suggest days of the week, months of the year, and so on.

- Can anyone suggest how a pattern might help us predict something?

  Answer: Answers will vary. Students may say that since we know the days of the week repeat in a pattern, we know how many days it will be until Saturday.

Read the instructions in setting the direction aloud to students as they read along.

Rule: Start with a square and a triangle.
Add a square and a triangle each time.

a. How many elements repeat? 2 elements: a square and a triangle

b. What is the 6th shape in the pattern? a triangle

c. Predict what the 12th shape would be in the pattern. Explain how you know.
The 12th shape would be a triangle, because every even-numbered shape is a triangle.

d. Predict what the 25th shape would be in the pattern. Explain how you know.
The 25th shape would be a square, because every odd-numbered shape is a square.

e. Explain how you would find the 100th shape in the pattern.
The 100th shape would be a triangle, because all even-numbered shapes are triangles.

Student pages 181–182

teaching strategies

Students may confuse *pattern* and *design*. Explain that a design may include a pattern, but it is not necessary. A pattern is a regular or ongoing repetition in a situation or in a design. Point out examples of designs that include patterns and ones that do not. Tell students that patterns are found in all areas of math. It is important to be able to identify, describe, and extend patterns.

Have students cut out all of the shapes on student page 185. Have students work solo to use the rule to create a pattern with the shapes.

- The repeating part of a pattern can be described by a rule. The rule can be used to continue the pattern. What is the rule for this pattern?

  Answer: Start with a square and a triangle, and add a square and a triangle each time.

- An element is one item in the pattern. The string of elements that repeat is called "the core." How many elements are in this pattern? What are the elements? What is the core?

  Answer: There are two elements: a square and a triangle. The core is square-triangle.

- What is the 6th shape in the pattern? Explain how you know.

SHAPES FOR PATTERNS

Student page 185

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Answer: The 6th shape is a triangle. Explanations may vary. Students may say they found the shape by counting.

- What is the 12th shape in the pattern? Explain how you know.

  Answer: The 12th shape is a triangle. Explanations may vary. Students may have extended the pattern and counted to find the 12th shape.

Encourage students to think of another way to find the 12th shape without extending the pattern and counting. Have students number the elements on the pattern on their student pages from 1–8. Explain that these numbers tell the position number of each shape.

- What do you notice about the numbers for the triangles?

  Answer: They are all even.

- Is 12 even or odd?

  Answer: 12 is even.

- So knowing that all triangles are even numbers, can you tell me what the 100th shape in this pattern is?

  Answer: The 100th shape is a triangle because all triangles are even-numbered shapes and 100 is even.

- What is the 25th shape in the pattern?

  Answer: The 25th shape is a square because 25 is odd and all odd-numbered shapes are squares.

teaching strategies

The rule in this problem was repeating square-triangle. After making their pattern, have students look for a different rule that might apply. Ask, "How else could we describe this pattern?" It is good for students to learn that often more than one rule can apply depending on what attribute is being identified. In this case, the pattern could also be a number of sides pattern (4 sides-3 sides). Point out that patterns like these are called AB patterns. If students do not know this convention, explain that a shortcut to describing patterns is to use letters to stand for different elements. Mathematicians usually use the first letters of the alphabet for this purpose.

english language learners

If time permits, have students create graphic organizers to help them remember and understand key vocabulary words.

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work time

Ask a student to read problem 1 aloud to the group.

Have students work solo on problem 1 until you tell them to switch to partner work.

After a few minutes, instruct students to share their work with partners. Tell students to explain their patterns and answers to their partners. Then have students edit their work based on their partners' feedback.

Observe students as they talk together.

- Are the listeners asking questions when the speaker is unclear?
- Do both students explain their solutions even if they did something similar?

work time

Use the given rule to make a pattern with the shapes.

Glue the shapes down when you are done.

1. Rule: Start with 2 circles and 1 triangle.
   Add 2 circles and 1 triangle each time.

a. How many elements repeat?

3 elements: 2 circles and 1 triangle

b. What is the 9th shape in the pattern?

a triangle

c. Predict what the 24th shape would be in the pattern. Explain how you know.

The 24th shape would be a triangle, because every third shape (3rd, 6th, 9th, ...) is a triangle.

d. Predict what the 35th shape would be in the pattern. Explain how you know.

It would be a circle. Since every third shape is a triangle, the 36th shape is a triangle. Every triangle is preceded by a circle, so the 35th shape is a circle.

e. Explain how you would find the 100th shape in the pattern.

It would be a circle. Since every third shape is a triangle, the 99th shape is a triangle. Every triangle is followed by a circle, so the 100th shape is a circle.

Student pages 182–183

english language learners

This is a good opportunity for ELL students to share ideas with each other by working cooperatively. This interaction helps students develop English naturally by interacting with others.

scaffolding for success

Encourage special needs students to ask their partners to help clarify any concepts or terminology they still find confusing.

ERROR

Look for students who do not understand how to predict a shape in the pattern. Help students see that if the position number is evenly divisible by 3, then the shape is a triangle because the triangle is every third shape. Students can count by 3s to find the position of all triangles. If the position number has a remainder when divided by 3, then the shape is a circle since the other two elements are circles.

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probing for understanding

For problem 1, ask questions such as the following:

- How many elements are there in this pattern?
  Answer: There are 3 elements: 2 circles and 1 triangle.
- What is the core, the part that repeats?
  Answer: The core is circle-circle-triangle.
- What is the 24th shape in the pattern? Explain how you know.
  Answer: The 24th shape is a triangle. Explanations may vary. Students may say that they counted to find the 24th shape or that all multiples of 3 are triangles or that if you divide the position number by 3, there is no remainder, so the shape is a triangle.
- How long can we continue this pattern?
  Answer: We can continue the pattern forever.
- How else could you describe this pattern?
  Answer: Answers will vary: 0-sides, 0-sides, 3-sides or AAB.
- Did anyone have trouble figuring out the 100th shape? Explain why.
  Answer: Answers will vary.

scaffolding for success

It is critical that students feel safe enough to share their ideas, strategies, and, most importantly, mistakes. Most students—and special needs students in particular—are not used to talking about mathematics; yet in higher-level courses, student success might depend on participation in study groups and the student's ability to share ideas.

work time

Ask a student to read problem 2 aloud to the group.

Have students work solo on problem 1 to start. After a few minutes, have students move to partner work.

As partners are discussing the pattern and their answers, spend time conferencing with one or two students.

2. Rule: Start with 1 square, 1 triangle, and 1 circle.
Add 1 square, 1 triangle, and 1 circle each time.

a. How many elements repeat? 3 elements: 1 square, 1 triangle, and 1 circle

b. What is the 8th shape in the pattern?
a triangle

c. Predict what the 36th shape would be in the pattern. Explain how you know.
The 36th shape would be a circle, because every third shape (3rd, 6th, 9th, ...) is a circle.

Student page 183

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english language learners

ELLs and other students may not easily recognize the progress they have made. Point out the observations you have made about the misconceptions they have overcome and the achievements they have made with their math skills, and with their math language and terms.

d. Predict what the 49th shape would be in the pattern. Explain how you know.

It would be a square. Since every third shape is a circle, the 48th shape is a circle. Every circle is followed by a square, so the 49th shape is a square.

e. Explain how you would find the 1,000th shape in the pattern.

Divide 1,000 by 3, and the result is 333 R1. That means that the 999th shape is a circle, and the following shape is a square. So the 1,000th shape is a square.

Student page 184

probing for understanding

For problem 2, ask questions such as the following:

- How many elements are there in this pattern?

  Answer: There are 3 elements: 1 square, 1 triangle and 1 circle.

- What is the core, the part that repeats?

  Answer: The core is square-triangle-circle.

- What is the 36th shape in the pattern? Explain how you know.

  Answer: The 36th shape is a circle. Explanations may vary. Students may say that they counted to find the 36th shape or that all multiples of 3 are circles or that if you divide the position number by 3, there is no remainder, so the shape is a circle.

- What is the 49th shape in the pattern? Explain how you know.

  Answer: The 49th shape is a square. Explanations may vary. Students may say that they counted to find the 49th shape or that the 48th shape is a circle and every circle is followed by a square or that $49 \div 3 = 16$ R1, so it is the first shape after the circle, which is the square.

- How else could you describe this pattern?

  Answer: Answers will vary: 4-sides, 3-sides, 0-sides or ABC.

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work time

Read all parts of problem 3 aloud to the group.

Have students work solo and then compare their explanations with their partners.

3. a. How are the patterns in problem 1 and problem 2 similar?

Possible answer: Both patterns have 3 elements that repeat. For both patterns, you can divide the position number by 3 to find what element is in that position.
The remainder tells you what the element is.

b. How are the patterns in problem 1 and problem 2 different?

Possible answer: The elements that repeat are different. In problem 1, the elements are 2 circles and 1 triangle. In problem 2, the elements are 1 square, 1 triangle, and 1 circle. Problem 1 is an AAB pattern, and problem 2 is an ABC pattern.

Student page 184

probing for understanding

Display both patterns for all students to see. Ask questions such as the following:

- What is similar about these two patterns?

  Answer: Answers will vary. Both patterns have 3 elements in the core that repeat. For both patterns you can divide the position number by 3 to find what shape is in that position. The remainder tells you what the shape is.

- What is different about these two patterns?

  Answer: Answers will vary. The core is different. In problem 1, the core is circle-circle-triangle. In problem 2, the core is square-triangle-circle. Problem 1 is an AAB pattern, and problem 2 is an ABC pattern.

- Can you name another ABC pattern that uses the same shapes as in problem 2?

  Answer: Answers will vary. Possible answer: circle-triangle-square.

english language learners

It is important to point to the visuals as you are describing similarities and differences. Pointing is a simple, yet effective strategy to help ELL students understand concepts. Have ELL students share their solutions on the board.

reflection

Writing a Reflection

When you have about 2 minutes left, stop the discussion, even if you are not finished.

Have students respond to the reflection prompt.

reflection

One thing I learned today about repeating shape patterns is …

Student page 187

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